MA 135: College Algebra (Fall 2016)
Instructor: Justin Ryan
Review for Midterm Exam 1

Butler Community College

How to use this handout—Complete all problems, showing enough work. Problems on the midterm exam will be similar to these in nature, but this review packet is **not** meant to be comprehensive. **You should still study your notes and review your homework**.

Important Announcement—You will not be allowed to use a calculator or any other electronic devices on the midterm exam. You will however be permitted to use one 3×5 in^2 note card of your own hand-written notes. If your paper is too large, I will cut it. If your notes are not hand-written (if there are pictures or typed portions), I will cut it.

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1. Give the equation of the line in slope-intercept form that passes through the point $(-6, 7)$ and is parallel to the line $-6x + 7y = 43$.

$P(-6,7)$

$7y = 6x + 43$

$y = \underbrace{\boxed{\tfrac{6}{7}}}_{=m} x + \frac{43}{7}$

$\rightarrow$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{6}{7}(x+6)$

$y - 7 = \frac{6}{7}x + \frac{36}{7}$

$y = \frac{6}{7}x + \frac{36}{7} + \frac{49}{7}$

$\boxed{y = \frac{6}{7}x + \frac{85}{7}}$

2. Give the equation of the line in slope-intercept form that passes through the point $(7, 3)$ and is perpendicular to $-7x + 3y = -58$.

$P(7,3)$

$3y = 7x - 58$

$y = \frac{7}{3}x - \frac{58}{3}$

so $m = \frac{-3}{7}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-3}{7}(x - 7)$

$y = \frac{-3}{7}x + \frac{3}{\cancel{7}}\frac{\cancel{7}}{1} + 3$

$y = \frac{-3}{7}x + 3 + 3$

$\boxed{y = \frac{-3}{7}x + 6}$

3. Find $f(0)$ given that

$$f(x)=\begin{cases} x-2 & \text{if } x<5 \\ 9-x & \text{if } x\geq 5 \end{cases}.$$

$0<5$, so $f(0)=0-2=\boxed{-2}$

4. Write the function in vertex form and describe how the graph of the function $y=f(x)$ can be obtained from that of the parent function $p(x)=x^2$.

$$f(x)=x^2+12x+32$$

$b=12$

$\frac{b}{2}=6$

$\left(\frac{b}{2}\right)^2=36$

$f(x)=(x^2+12x+36)+32-36$

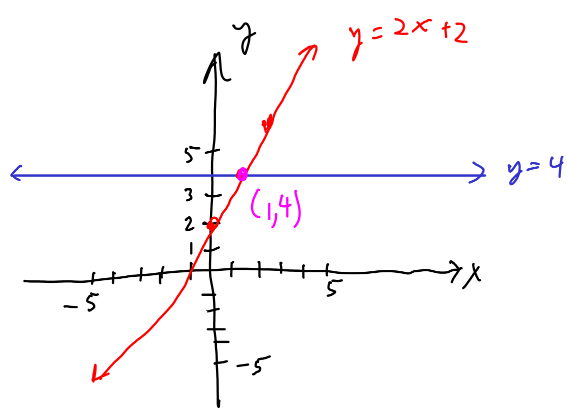
$\boxed{f(x)=(x+6)^2-4}$

shift the graph $y=x^2$ left 6 and down 4.

5. Solve the inequality by graphing $2x+2\geq 4$.

$y=2x+2$

$y=4$



The red graph is above the blue graph to the right of the intersection, so the solution is

$\boxed{x\geq 1}$

6. Determine whether the function is even, odd, or neither. What kind of symmetry does the graph of $y = f(x)$ have, if any?

$$f(x) = 8x^{4} - 3x^{2} + 3x^{0}$$

All exponents are even, so by inspection f is even.

The graph of $y=f(x)$ thus has symmetry about the y-axis.

7. Find the distance between the points $P(-4, 4)$ and $Q(6, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-(-4))^2 + (-3-4)^2} = \sqrt{10^2 + (-7)^2} = \boxed{\sqrt{149}}$$

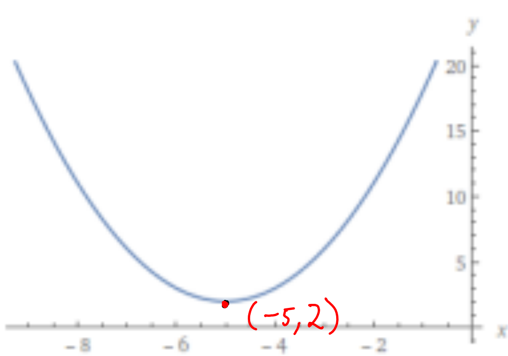
8. Find the midpoint of the line segment connecting the points $P(-4, 4)$ and $Q(6, -3)$.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-4+6}{2}, \frac{4-3}{2}\right)$$

$$\boxed{M = \left(-1, \tfrac{1}{2}\right)}$$

9. The following graph defines a function, $y = f(x)$. Use the graph to identify the parent function and all translations, then write an equation of the function. What are the domain and range of f?



Parent function: $p(x) = x^2$

Transformations: left 5, up 2

Equation: $\boxed{f(x) = (x+5)^2 + 2}$

Domain: $\mathbb{R}$

Range: $[2, \infty)$

10. The graph of $y = x^2$ is shifted 4 units to the left. This graph is then vertically stretched by a factor of 6 and reflected across the x-axis. Finally, the graph is shifted 7 units downward. Give an equation of the function defined by the resulting graph.

$$\boxed{f(x) = -6(x+4)^2 - 7}$$

11. Graph the line. Clearly label the x- and y-intercepts (if they exist) and identify the slope of the line.

$$f(x) = -\tfrac{1}{2}x + 2$$

y-intercept: $(0,2)$

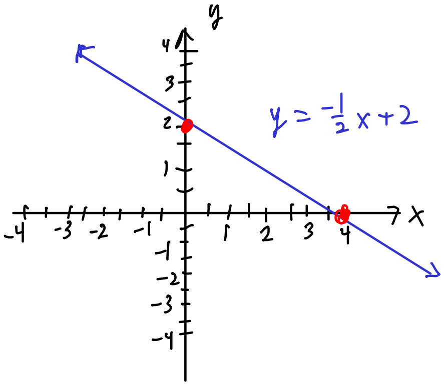
x-intercept: $(4,0)$

$0 = -\frac{1}{2}x + 2$

$\frac{1}{2}x = 2$

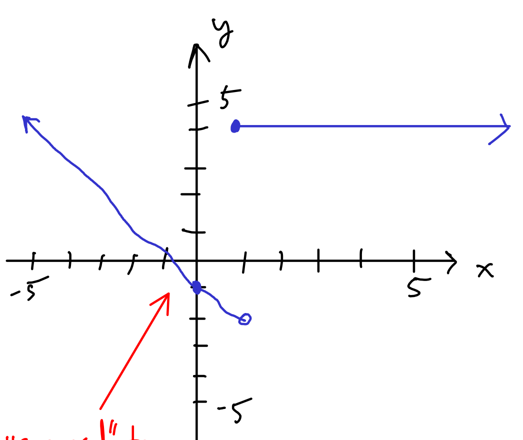
$x = 4$

Slope: $m = -\frac{1}{2}$



12. Sketch a graph of the function. On what intervals is f increasing, decreasing, and constant?

$$f(x) = \begin{cases} -4 & \text{if } x \geq 1 \\ -1 - x & \text{if } x < 1 \end{cases}$$



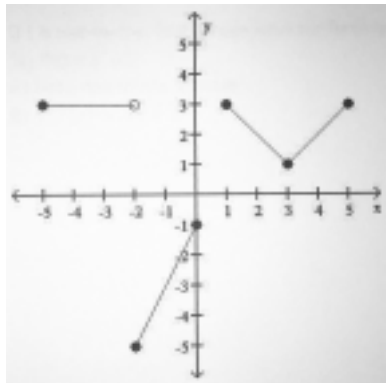
"supposed" to be a line "

increasing: None

decreasing: $(-\infty, 1)$

constant: $(1, \infty)$

13–14. For problems 13 and 14, refer to the graph.



13. On what intervals is the function increasing, decreasing, and constant?

Increasing: $(-2,0) \cup (3,5)$

Decreasing: $(1,3)$

Constant: $(-5,-2)$

14. At what points is the graph discontinuous? What types of discontinuities does the graph have?

At $x=-2$ the graph has a jump discontinuity.

From $x=0$ to $x=1$ the graph is not defined, so we could regard this as an interval jump discontinuity.

15. Determine the domain of the function. Identify and classify all points of discontinuity.

$$f(x) = \frac{x^3-8}{x-2}$$

Domain: $x \neq 2$, continuous elsewhere.

$$f(x) = \frac{x^3-8}{x-2} = \frac{x^3-2^3}{x-2} = \frac{\cancel{(x-2)}(x+2x+4)}{\cancel{x-2}} = x+2x+4, \ x \neq 2.$$

So at $x=2$, the function has a hole.

16. Determine the domain of the function. Identify and classify all points of discontinuity.

$$f(x) = \frac{x^2+2x+1}{x^2-1}$$

Domain : $x \neq -1, +1$, continuous elsewhere

$$f(x) = \frac{x^2+2x+1}{x^2-1} = \frac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}(x-1)} = \frac{x+1}{x-1}, \; x \neq -1$$

So f has a hole at $x=-1$ and a vertical asymptote at $x=+1$.

17. Write the function in vertex form and clearly identify all transformations of the parent graph, in the proper order.

$$f(x) = -2x^2 + 6x - \tfrac{13}{2}$$

$$f(x) = -2\left(\left(x^2 - 3x + \tfrac{9}{4}\right) + \tfrac{13}{4} - \tfrac{9}{4}\right)$$

$b = -3$

$\left(\frac{b}{2}\right) = \frac{-3}{2}$

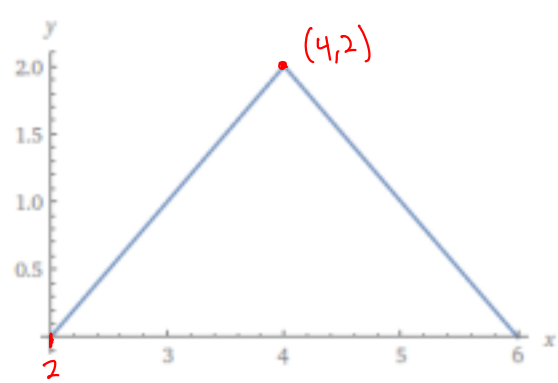
$\left(\frac{b}{2}\right)^2 = \frac{9}{4}$

$$= -2\left(\left(x - \tfrac{3}{2}\right)^2 + \tfrac{4}{4}\right)$$

$$\boxed{f(x) = -2\left(x - \tfrac{3}{2}\right)^2 - 2}$$

Transformations:
1. vertical stretch by 2
2. Reflect over x-axis
3. Shift right by 3/2
4. shift down 2

18. The following graph defines a function, $y = f(x)$. Use the graph to identify the parent function and all transfomations (in order), then write an equation of the function. What are the domain and range of f?



Weird scale!
I won't do this on the exam. Sorry!

Parent : $p(x) = |x|$

Transformations : Right 4
Reflect over x-axis
up 2

Equation : $\boxed{f(x) = -|x-4| + 2}$

Domain : $\mathbb{R}$

Range : $(-\infty, 2]$