MA 135: College Algebra (Fall 2016)
Instructor: Justin Ryan
Review for Midterm Exam 2

Butler Community College

How to use this handout—Complete all problems, showing enough work. Problems on the midterm exam will be similar to these in nature, but this review packet is **not** meant to be comprehensive. **You should still study your notes and review your homework and Good Problems.**

Important Announcement—You will not be allowed to use a calculator or any other electronic devices on the midterm exam. You will however be permitted to use one 3×5 in^2 note card of your own hand-written notes. If your paper is too large, I will cut it. If your notes are not hand-written (if there are pictures or typed portions), I will cut it.

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1. A projectile is thrown upward so that its distance above the ground after t seconds is

$$h(t) = -10t^2 + 240t \text{ feet.}$$

After how many seconds does it reach its maximum height? What is the maximum height?

$V = \left(\frac{-b}{2a}, h\left(\frac{-b}{2a}\right)\right) = \left(\frac{-240}{2(-10)}, h\left(\frac{-240}{2(-10)}\right)\right) = (12, h(12))$

$h(12) = -10(12)^2 + 240(12)$

$= -10(144) + 2(120)(12) = -1440 + 2(1440) = 1440$ ft

It takes 12 seconds for the projectile to reach its maximum height of 1440 ft.

2. Solve the equation $|3x + 6| - 2 = 7$.

$|3x+6| = 9$

$3x+6 = 9$

$3x = 3$

$x = 1$

$3x+6 = -9$

$3x = -15$

$x = -5$

3. Let $f(x) = 9x - 3$ and $g(x) = 2x + 8$. Find a formula for fg.

$(fg)(x) = (9x-3)(2x+8)$

$= 18x^2 + 72x - 6x - 24$

$\boxed{(fg)(x) = 18x^2 + 66x - 24}$

4. Let $f(x) = \sqrt{x+3}$ and $g(x) = 8x - 7$.

(*a*) Find a formula for $f \circ g$.

$(f \circ g)(x) = f(g(x))$

$= f(8x-7)$

$= \sqrt{(8x-7)+3}$

$= \sqrt{8x-4}$

$= \sqrt{4}\sqrt{2x-1}$

$\boxed{(f \circ g)(x) = 2\sqrt{2x-1}}$

(*b*) What is the domain of $f \circ g$?

$D_g = \mathbb{R}$ so the restriction is totally in f.

$2x - 1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$\boxed{D_{f \circ g} = [\frac{1}{2}, \infty)}$

5. Solve the inequality and plot your solution on a number line.

$$|x-5| - 9 > 9$$

First, set it equal.

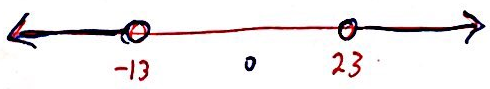
$|x-5| - 9 = 9$

$|x-5| = 18$

$x - 5 = 18$ $\quad$ $x - 5 = -18$

$x = 23$ $\quad$ $x = -13$

Next, plot on number line



Test $x = 0$: $|0-5| - 9 = 5 - 9 = -4 > 9$ ✗ No.

So, shade outside.

$\boxed{\text{Solution: } (-\infty, -13) \cup (23, \infty)}$

6. State whether the vertex of the function is a maximum or minimum, and give the corresponding maximum or minimum value of the function.

$$f(x) = x^2 + 12x + 32$$

Vertex is a minimum since $a > 0$.

$x = \frac{-b}{2a} = \frac{-12}{2(1)} = -6$

$f(-6) = (-6)^2 + 12(-6) + 32 = 36 - 72 + 32 = -4$

The minimum value of the function f is $f(-6) = -4$.

7. Identify the vertex of the parabola.

$$P(x) = 3x^2 - 18x + 31$$

$x = \frac{-b}{2a} = \frac{-(-18)}{2(3)} = \frac{+18}{6} = 3$

$y = P(3) = 3(3^2) - 18(3) + 31$

$= 27 - 2(27) + 31$

$= 31 - 27 = 4$

Vertex = $(3, 4)$

8. Solve the inequality analytically. Support your answer graphically. Give exact values for the endpoints.

$$x^2 + 5x \le -6$$

$x^2 + 5x + 6 \le 0$

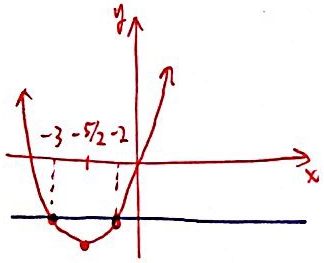
$(x+3)(x+2) \le 0$

$x = -3, -2$

Soln: $[-3, -2]$

Test $x = 0$: $0^2 + 5 \cdot 0 \le -6$ ✗ No!

$x^2 + 5x = (x + \frac{5}{2})^2 - \frac{25}{4}$



9. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$, for the function f and completely simplify.

$$f(x) = 4x - 12$$

$f(x+h) = 4(x+h) - 12$

$= 4x + 4h - 12$

$- \quad f(x) = 4x - 12$

$f(x+h) - f(x) = 4h$

$h \qquad h$

$\Rightarrow \quad \frac{f(x+h)-f(x)}{h} = 4$

10. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$, for the function f and completely simplify.

$$f(x) = 3x^2 - 6x$$

$f(x+h) = 3(x+h)^2 - 6(x+h)$

$= 3x^2 + 6xh + 3h^2 - 6x - 6h$

$- \quad f(x) = 3x^2 - 6x$

$f(x+h) - f(x) = 6xh - 6h + 3h^2$

$h \qquad h$

$\frac{f(x+h)-f(x)}{h} = 6x - 6 + 3h$

11. Divide as indicated. Write the quotient in standard form.

$$\frac{(3-9i)}{(5-7i)}\frac{(5+7i)}{(5+7i)} = \frac{15-45i+21i-63i^2}{5^2+7^2} = \frac{78-24i}{74} = \frac{78}{74} - \frac{12}{37}i = \boxed{\frac{39}{37} - \frac{12}{37}i}$$

12. Solve the inequality analytically, writing the solution set in interval notation.

$$8 < \frac{3x-10}{3} < 13$$

$$24 < 3x - 10 < 39$$

$$34 < 3x < 49$$

$$\frac{34}{3} < x < \frac{49}{3}$$

$$\boxed{\left(\frac{34}{3}, \frac{49}{3}\right)}$$

13. Find $(f+g)(4)$ where $f(x) = x-1$ and $g(x) = x+2$.

$$(f+g)(4) = f(4) + g(4)$$
$$= (4-1) + (4+2)$$
$$= 3 + 6$$
$$\boxed{(f+g)(4) = 9}$$

14. Solve each equation or inequality.

(*a*) $|4x+8| = 4$

$4x+8 = 4$ $\qquad$ $4x+8 = -4$

$4x = -4$ $\qquad$ $4x = -12$

$\boxed{x = -1 \qquad x = -3}$

(*b*) $|4x+8| < 4$

$\boxed{(-3,-1)}$

Test $x=0$: $|4\cdot 0+8| = 8 < 4$ ✗ No!

So, shade inside

(*c*) $|4x+8| > 4$

This one is opposite, so shade outside to get

$\boxed{(-\infty,-3) \cup (-1,\infty)}$

15. Consider the function $P(x) = 2x^2 + 8x + 5$.

(*a*) Write P in vertex form.

$P(x) = 2\left(x^2 + 4x + 4 + \frac{5}{2} - 4\right)$

$= 2\left((x+2)^2 - \frac{3}{2}\right)$

$\boxed{P(x) = 2(x+2)^2 - 3}$

(*b*) Identify the vertex and axis of symmetry.

$\boxed{\text{Vertex} = (-2,-3)}$

$\boxed{\text{Axis}: \quad x = -2}$

(*c*) State the domain and range of P.

$\boxed{\text{Domain} = \mathbb{R} = (-\infty,\infty)}$

$\boxed{\text{Range} = [-3,\infty)}$