1. Limits

These Good Problems cover material from section 2.2 of our book. These good problems consist of a mix of computational and conceptual exercises. The goal is to cement the ideas of this week's lecture, with an eye looking forward to next week's lecture.

1. Consider the function $f(x) = \dfrac{x^2 + 3x + 3}{x - 2}$. Compute the following limits, if they exist.

a.) $\lim_{x \to 1} f(x)$

b.) $\lim_{x \to 2} f(x)$

2. Consider the function $f(x) = \dfrac{2x^2 - x - 1}{x - 1}$.

a.) What is the domain of f?

b.) Calculate $\lim_{x \to 1} f(x)$.

3. Consider the function $f(x) = \dfrac{x+7}{x^2+9x+14}$.

a.) Compute $\lim\limits_{x\to -7} f(x)$

b.) Compute $\lim\limits_{x\to -2} f(x)$

Use a graphing utility to graph the function $y = f(x)$. Explain in terms of the graph of $y = f(x)$ why the limits $\lim\limits_{x\to -7} f(x)$ and $\lim\limits_{x\to -2} f(x)$ are fundamentally different.

4. Consider the function $f(x) = x^2$.

a.) What is $f(2)$?

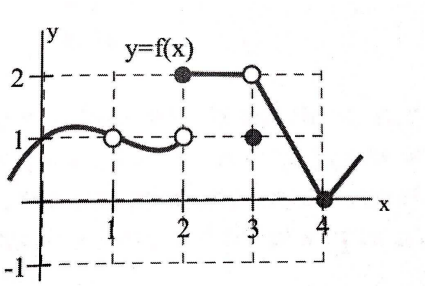
b.) Compute $\lim_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

5. Consider the function $f(x) = x^6$.

a.) Compute the difference quotient of f, $DQ = \dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$. [Hint: Use Pascal's Triangle (Binomial Theorem) to compute $f(x+h)$.]

b.) Compute $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

6. Consider the following graph.



Compute the following,

i.) $f(1) =$

ii.) $\lim_{x\to 1} f(x) =$

iii.) $f(3) =$

iv.) $\lim_{x\to 3} f(x) =$

v.) $f(2) =$

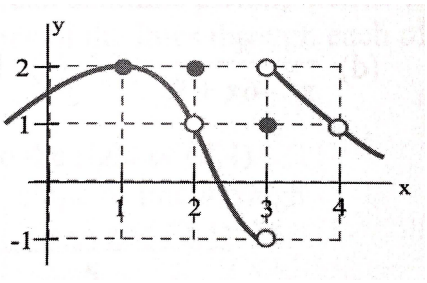
vi.) $\lim_{x\to 2^-} f(x) =$

vii.) $\lim_{x\to 2^+} f(x) =$

viii.) $\lim_{x\to 2} f(x) =$

ix.) At what x-values is the function discontinuous?

7. Consider the following graph.



Compute the following,

i.) $f(2) =$

ii.) $\lim_{x\to 2} f(x) =$

iii.) $f(4) =$

iv.) $\lim_{x\to 4} f(x) =$

v.) $f(3) =$

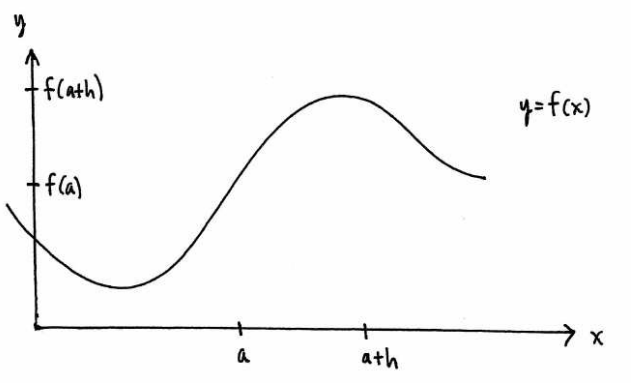
vi.) $\lim_{x\to 3^-} f(x) =$

vii.) $\lim_{x\to 3^+} f(x) =$

viii.) $\lim_{x\to 3} f(x) =$

ix.) At what x-values is the function discontinuous?

8. Consider the following graph of a function $y = f(x)$. Here a is a point in the domain of f and h is a nonzero constant.



a.) Sketch the secant line passing through the points $(a, f(a))$ and $(a + h, f(a + h))$.

b.) Write an expression that represents the slope of this secant line?

c.) Suppose you take the limit as $h \to 0$. Describe what happens to the secant line that you drew in part *a*.

d.) Describe what the limit of the slope in part *b* represents geometrically.

9. Compute $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for the function $f(x) = \sqrt{x}$.

10. Compute $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for the function $f(x) = \dfrac{1}{x}$.