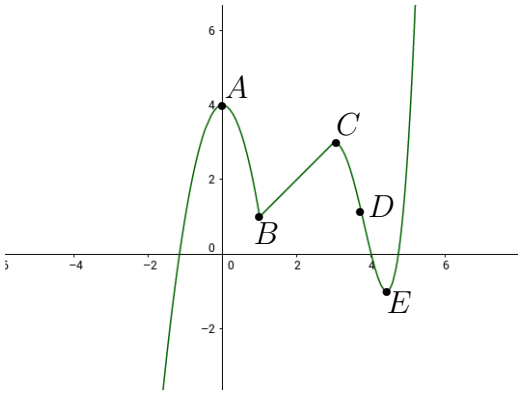
4. Derivative Tests, and Review

These Good Problems study the concepts presented in sections 2.6 and 2.7 of our text, then review material from the entire semester thus far as preparation for the first midterm exam.

1. Find the intervals of concavity of the function $f(x) = 4x^4 - 8x^2 + 5$.

2. Determine the concavity of the function $y = x\ln(x)$ at $x = \frac{1}{e}$.

3. Determine which points on the graph are local extrema, and which are inflection points.



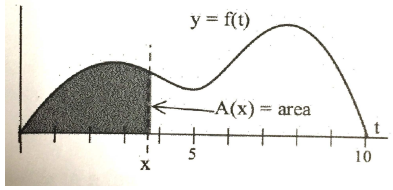
4. Determine the absolute (global) maximum and/or minimum values of the function $f(x) = \dfrac{1}{x^2+1}$, if they exist.

5. Find all critical points and local extreme points of the function

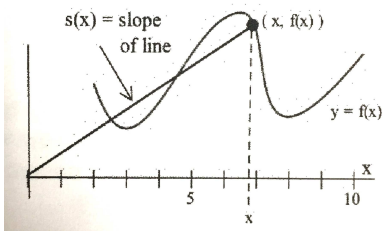
$$f(x) = \ln(x^2 - 6x + 11),$$

and determine whether the extreme points are maxima or minima.

6. let f be a non-negative function on the interval $[0, 10]$ such that $f(0) = f(10) = 0$. Define $A(x)$ to be the area bounded between the x-axis, the graph of $y = f(t)$, and the vertical line at x. At what value of x is $A(x)$ a maximum? At what value of x is $A(x)$ a minimum?



7. Let f be a positive function on the interval $[2, 10]$ with graph shown below. Define $S(x)$ to be the slope of the line through the points $(0, 0$ and $(x, f(x))$. At what value of x is $S(x)$ a maximum? At what value of x is $S(x)$ a minimum?



The remaining questions are intended to be review for the first midterm exam.

8. Compute the limits. Be sure to show enough work and use proper notation.

a.) $\lim_{x \to 4} 3x^2 - 9x + 4$

b.) $\lim_{t \to 0} \dfrac{t^2 - t}{2t}$

c.) $\lim_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$

d.) $\lim_{x \to -2} \dfrac{x^5 + 32}{x + 2}$

e.) $\lim_{u \to 0^+} \ln u$

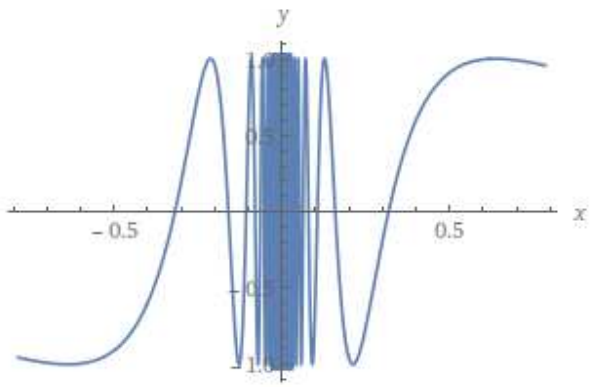
9. Use the *limit definition of derivative* to compute $\frac{dy}{dx}$ for $y = x^2 + 2x$. You **must** use the limit definition and show enough work to receive any credit.

10. Use the *limit definition of derivative* to compute $\frac{dy}{dx}$ for $y = x^6$. You **must** use the limit definition and show enough work to receive any credit.

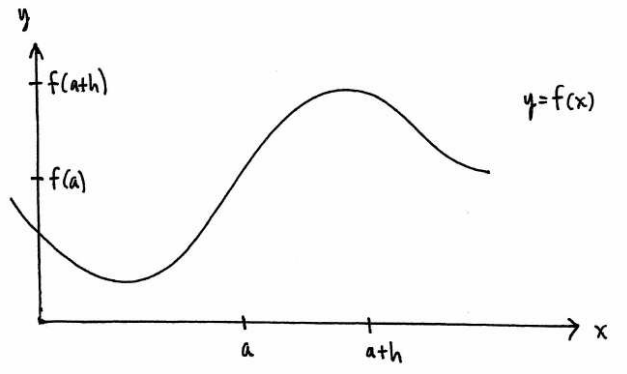
11. Find an equation of the tangent line to the curve $y = \ln(x)$ when $x = 1$.

12. Find all points on the curve $y = e^{6x^3-9x^2}$ at which the tangent line is horizontal.

13. Consider the graph of the function $y = f(x)$ given below. Explain why the limit as x approaches 0 does not exist.



14. Use the graph below to describe how the derivative of a function is defined at a point in terms of the secant lines through the point.



15. Find an equation of the normal line to the graph of $y = x^3$ at the point $(-2, -8)$. (*The normal line is perpendicular to the tangent line.*)

16. Compute the derivatives. You can use the "shortcut" rules, but be sure to show enough work.

a.) $f(x) = 2^x(x^2 - 5x)$

b.) $y = te^{t^2}$

c.) $x(t) = \ln(t^2 - 1)$

d.) $g(x) = \dfrac{\ln x}{x^2 - 1}$

e.) $q(s) = (s^3 - 9)^4$

f.) $p(x) = 6x^5 - \frac{15}{2}x^4 + 10x^3 - 15x^2 + 30x - 3000$

17. State the (limit) definition of continuity.

18. Determine whether the function is continuous. If it is discontinuous, state where.

$$f(x) = \begin{cases} x^2 & x \leq 2, \\ 6 - x & 2 < x < 6, \\ 2x - 17 & x \geq 6. \end{cases}$$

19. Determine whether the function is continuous. If it is discontinuous, state where.

$$g(x) = \frac{12}{5x^3 - 5x}$$

Some Comments

This review is not meant the be comprehensive. You should also study past Good Problems and Recommended Exercises.

The exam will be structured as follows. There will be 5 True/False questions and 5 "Fill in the Blank" questions, each worth 1 point each. Then there will be 18 Multiple Choice questions and 2 Short Answer questions, each worth 5 points each. The Multiple Choice questions are all or nothing (no partial credit), but partial credit will be possible on the Short Answer questions.

You will *not* be allowed to use a calculator or any other electronic device on the exam. You will be allowed to use a single 3×5 in^2 note card of your own hand-written notes. If the note card is too big, or if the notes are not written by hand, then you will not be allowed to use the note card on the exam.

You'll also need to know...

Definitions!

I won't ask you to state any definitions word-for-word, but I will expect you to know them. Definitions are the most important part of this course. Our goal is to eventually use Calculus to help us solve real world problems. But we cannot use Calculus for anything if we don't know what the terms mean.