6. Related Rates Applications

In these Good Problems we will apply the idea of related rates to solve some real world problems.

For each of these problems, you should begin by drawing a picture (if appropriate), write down any equations that are not given, then do the calculus.

1. A right cylindrical tank is filled with water. The tank stands 2 meters tall and has a radius of 20 cm. How fast does the height of water in the tank drop when the water is being drained at a rate of 25 $\frac{\text{cm}^3}{\text{sec}}$? Find the exact rate that the height is decreasing when the height of the water is 1 meter high.

[Volume of a cylinder: $V = \pi r^2 h$.]

2. A storage bin is in the shape of a right circular cone. The bin stands 10 feet tall and the top rim of the bin has a radius of 4 feet. If a grain is being pumped into the bin at a rate of 2 cubic feet per minute, what is the rate of change of the depth of the grain when the depth is 5 feet?

[Volume of a cone: $V = \frac{1}{3}\pi r^2 h$.]

3. A company that manufactures pet toys calculates that its cost and revenue can be modeled by the functions

$$\begin{aligned} C(x) &= 50000 + 3.1x \\ R(x) &= 300x - 0.2x^2 \end{aligned}$$

where x represents the number of toys produced in a week. If production in one particular week is increasing at a rate of 300 toys per week, find:

a.) the rate at which the cost is changing,

b.) the rate at which the revenue is changing,

c.) the rate at which the profit is changing.

4. A company's profit is increasing at a rate of \$6000 per week. The demand and cost functions are given by

$$\begin{aligned} p(x) &= 5000 - 30x, \\ C(x) &= 3000x + 6000. \end{aligned}$$

Find the rate of change of sales $\left(\frac{dx}{dt}\right)$ when the weekly sales are $x = 50$ units.

5. A young woman and her boyfriend plan to elope, but she must first rescue him from his mother who has locked him in his room. The woman has placed a 20 foot long ladder against his house and is knocking on his window when his mother begins pulling the bottom of the ladder away from the house at a rate of 3 feet per second. How fast is the top of the ladder (including the young couple!) falling when the bottom of the ladder is

a.) 12 feet from the base of the wall?

b.) 16 feet from the base of the wall?

c.) 19 feet from the base of the wall?

6. Two resistors are connected in parallel with resistances R_1 and R_2 measured in Ohms (Ω). The total resistance, R, is given implicitly by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose that R_1 is increasing at a rate of $0.4\,\frac{\Omega}{\text{min}}$ and R_2 is decreasing at a rate of $0.7\,\frac{\Omega}{\text{min}}$. At what rate is R changing when $R_1 = 80\,\Omega$ and $R_2 = 105\,\Omega$?