7. Linear Approximations and Differentials

This week's Good Problems cover the linearizations of smooth functions. This topic is not in our book, but you can find my notes online at http://geometerjustin.com/teaching/bc/notes/.

1. Find the differential of each function.

a.) $y = x^2 e^x$

b.) $y = \dfrac{x}{1+2x}$

c.) $y = \sqrt{4+5x}$

d.) $y = \dfrac{1}{1+x}$

2. Find the linearization $y = L_a f(x)$ of the given function at the given value of a.

a.) $f(x) = x^4 + 3x^2, \quad a = -1$

b.) $f(x) = \dfrac{1}{\sqrt{2+x}}, \quad a = 0$

c.) $f(x) = x^{3/4}, \quad a = 16$

3. Use a linear approximation or differentials to approximate $(2.001)^5$.

4. Use a linear approximation or differentials to approximate $\dfrac{1}{1002}$.

5. Verify that the linearization of $f(x) = \sqrt[3]{1-x}$ at $a = 0$ is given by $L_0 f(x) = 1 - \frac{1}{3}x$.

For what values of x is $L_0 f$ accurate to within 0.1?

6. Consider the function $y = \sqrt{x}$. Compare the values of dy and Δy when $x = 1$ and $dx = \Delta x = 1$. Sketch a diagram of two triangles with segment lengths dx, dy, and Δy.

7. The edge of a cube is measured to be 30 cm, with a possible error of 0.1 cm. Use differentials to estimate the maximum possible error and relative error in computing the surface area of the cube.

8. The circumference of a sphere was measured to be 84 cm with a possible error of 0.5 cm. Use differentials to estimate the maximum error in the calculated volume. What is the relative error?

9. When blood flows along a blood vessel, *Poiseuille's Law* states that the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

$$F = kR^4.$$

A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery to widen it and restore the normal blood flow.

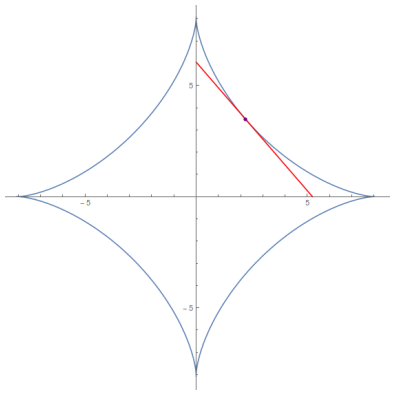
Show that the relative change in F is about four times the relative change in R. How will a 5% increase in the radius affect the flow of blood?

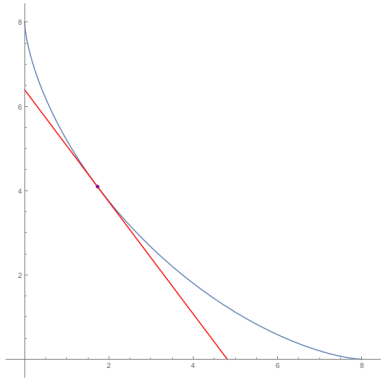
10. Suppose that you don't have a formula for f, but you know that $f(2) = -4$ and $f'(x) = \sqrt{x^2 + 5}$ for all x.

Use a linear approximation to estimate $f(1.95)$ and $f(2.05)$.

Are your estimates too large or too small? Explain.

11. Show that the length of the portion of any tangent line to the astroid $x^{2/3} + y^{2/3} = 4$ cut off by the coordinate axes is constant.





You can also find a link on my web page to a version of this graph with a slider to move the point of tangency along the curve:
https://geometerjustin.com/teaching/bc/gp/