Name:______________________________

M242: Calculus I (Fall 2018)

Instructor: Justin Ryan

Final Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any electronic devices. You may use a single two-sided 8.5 by 11 inch page of your own hand-written notes.

Part I: True/False [2 points each]

*Neatly write **T** if the statement is always true, and **F** otherwise.*

______**1.** Let f be a function satisfying $f(a) = k$. Then $\lim_{x\to a} f(x) = k$.

______**2.** If f is differentiable at $x = a$, then f is continuous at $x = a$.

______**3.** Suppose f' exists. The domain of f' coincides with the domain of f.

______**4.** If f and g are increasing on (a, b), then fg is increasing on (a, b).

______**5.** If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

______**6.** If f and g are continuous on $[a, b]$, then $\int_a^b \left[f(x) + g(x)\right] dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$.

______**7.** Every continuous function has a continuous antiderivative.

______**8.** Let f be a continuous function on the interval $[a, b]$. Then there exists a number c in $[a, b]$ such that $f(c) = f_{\text{avg}}$.

______**9.** If f is a continuous function on $[a, a+h]$, then $\lim_{h\to 0^+} f_{\text{avg}} = f(a)$.

______**10.** $\dfrac{d}{dx}\left[10^x\right] = x10^{x-1}$

Part II: Computations [5 points each]

Compute the following limits, derivatives, and integrals. Show enough work. Partial credit will be given when deserved.

11. Compute $\lim_{x \to 3} \dfrac{x^2 + 2x - 5}{x + 2}$.

12. Compute $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$.

13. Compute the derivative of $g(x) = x^2 \sin(\pi x)$.

14. Compute the derivative of $f(t) = \dfrac{t^4 - 1}{t^4 + 1}$.

15. Compute the derivative of $f(x) = x^2\sqrt{x^2+1}$.

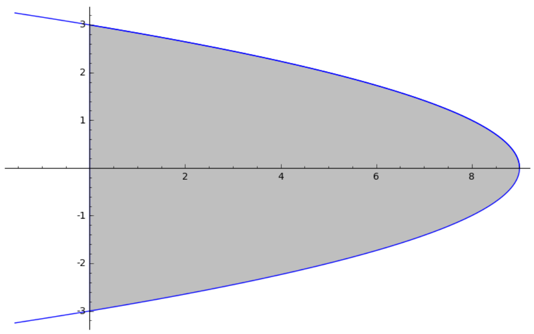
16. Find the particular antiderivative of $f(x) = \sin x + x$ satisfying $F(0) = 2$.

17. Compute the integral $\displaystyle\int_0^1 \frac{\left(\arcsin(x)\right)^3}{\sqrt{1-x^2}}\,dx$.

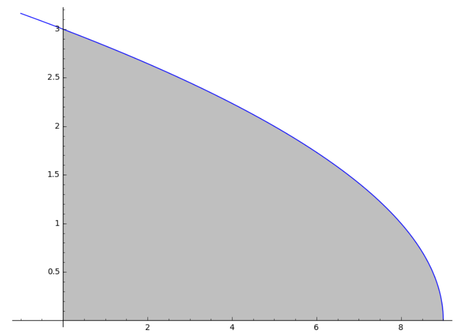
18. Compute the integral $\displaystyle\int \frac{x}{x^2+1}\,dx$.

19–21. Consider the region $\mathscr{R}$ bounded between the curves $x = 0$ and $x = 9 - y^2$, and the solid region $\mathscr{S}$ obtained by rotating $\mathscr{R}$ about the line $x = -1$.

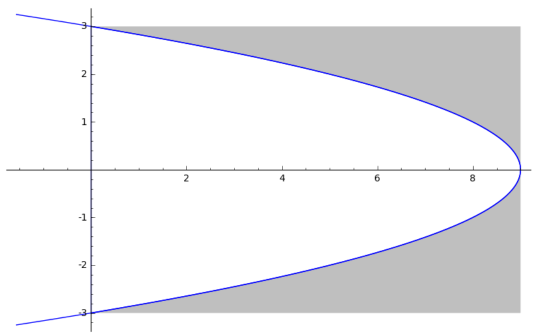
______**19.** Which graph best represents the region $\mathscr{R}$?



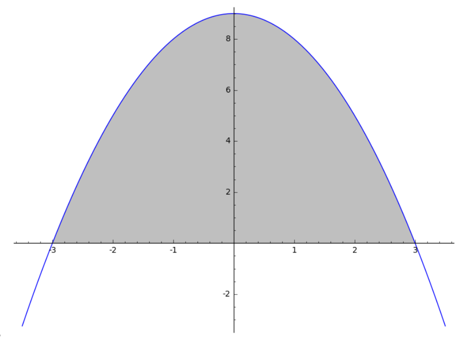
A.



B.



C.



D.

______**20.** Which integral represents the volume of the solid $\mathscr{S}$ using the slicing method?

A. $\pi \int_0^9 9 - x \, dx$

B. $\pi \int_{-3}^{3} \left((10 - y^2)^2 - 1\right) \, dy$

C. $\int_0^9 2\pi x \sqrt{9 - x} \, dx$

D. $2 \int_0^9 2\pi (x + 1) \sqrt{9 - x} \, dx$

______**21.** Which integral represents the volume of the solid $\mathscr{S}$ using the method of cylindrical shells? (Hint: Use symmetry.)

A. $\pi \int_{-3}^{3} \left((10 - y^2)^2 - 1\right) \, dy$

B. $\pi \int_0^9 9 - x \, dx$

C. $2 \int_0^9 2\pi (x + 1) \sqrt{9 - x} \, dx$

D. $\int_0^9 2\pi x \sqrt{9 - x} \, dx$

22. Find y' if $xe^y = y - 1$.

23. Use the limit definition of derivative to compute $f'(x)$. You must use the limit definition to receive credit.

$$f(x) = x^2 + 1$$

24. Compute $\int_0^2 x^3\,dx$ using the Riemann sum definition. You must use the definition to receive any credit.

25. Consider the function $f(x) = \dfrac{1}{\sqrt{x}}$ on the interval $[1,4]$.

a.) Verify that the Mean Value Theorem for Integrals applies to f.
b.) Find the value c guaranteed by the MVT.

26. Find two positive numbers x and y satisfying $x + 4y = 100$, such that their product is a maximum.

27. Let f be continuous on the interval $[0,1]$. Prove that $\int_0^1 f(x)\,dx = \int_0^1 f(1-x)\,dx$.

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