Name:

M242: Calculus I (Fall 2017)

Instructor: Justin Ryan

Chapter 3 Exam



Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]

Neatly write **T** if the statement is always true, and **F** otherwise.

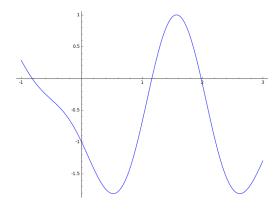
- _____1. If f'(c) = 0, then f has a local maximum of minimum at c.
- **____2.** If f has a local maximum of minimum at c, then f'(c) = 0.
- **____3.** If f and g are increasing on (a, b), then fg is increasing on (a, b).
- **____4.** If f' is continuous and $f'(x) \neq 0$ for all x, then $f(0) \neq f(1)$.
- **____5.** If f''(2) = 0, then (2, f(2)) is an inflection point of the curve y = f(x).

Part II: Multiple Choice [5 points each]

Compute the derivatives of the given functions. Select the best answer and write its corresponding letter neatly on the given line.

Consider the graph of a function y = f(x) below. You wish to use Newton's method to approximate the middle root. What should your initial guess be?

1



A.
$$x_0 = -1$$

B.
$$x_0 = 0$$

C.
$$x_0 = 1$$

D.
$$x_0 = 1.5$$

- **7–14.** Consider the function $f(x) = \frac{\sqrt{x^2 1}}{x 1}$.
- **____7.** What is the domain of f?
 - A. [-1, 1)

B. $(-\infty, -1] \cup (1, \infty)$

 \mathbf{C} . $(-\infty, -1) \cup (1, \infty)$

D. All real numbers

- **____8.** Compute $\lim_{x\to\infty} f(x)$.
- **A.** 1

 $B. +\infty$

C. -1

 \mathbf{D} . $-\infty$

- **____9.** Compute $\lim_{x \to -\infty} f(x)$.
- **A.** 1

 $B. +\infty$

C. -1

 \mathbf{D} . $-\infty$

- _____**10.** Compute $\lim_{x \to -1^-} f(x)$.
- $A. +\infty$

B. 0

C. −∞

D. $-\frac{1}{2}$

- _____11. Compute $\lim_{x\to 1^+} f(x)$.
- $A. +\infty$

B. 0

C. $-\infty$

D. 1

- **Recall, 7–14.** Consider the function $f(x) = \frac{\sqrt{x^2 1}}{x 1}$.
- **____12.** On what interval(s) is f increasing?
 - **A.** $(-\infty, -1) \cup (1, \infty)$

B. $(-\infty, -1)$

 \mathbf{C} . $(1,\infty)$

- **D.** Never increasing
- **____13.** On what interval(s) is the graph of f concave up?
 - **A.** $(-\infty, -1) \cup (1, \infty)$

B. $(-\infty, -1)$

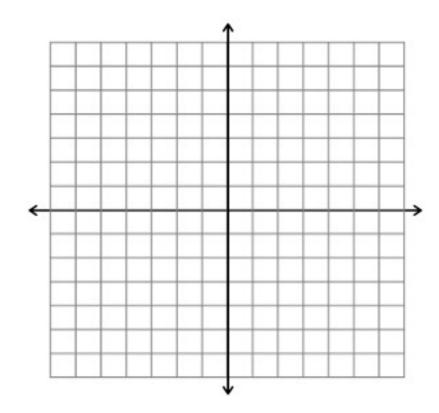
 $C.(1,\infty)$

D. Never concave up

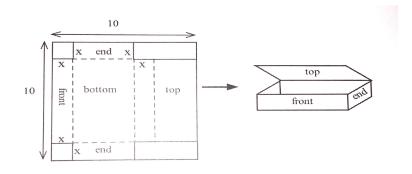
Part III: Written Problems [10 points each]

Complete all problems, showing enough work.

14. Sketch the graph of $f(x) = \frac{\sqrt{x^2 - 1}}{x - 1}$. Include labels and all other pertinent information.



15. You have a 10 inch by 10 inch piece of cardboard which you plan to cut and fold as show in the figure to form a box with a top. Find the dimensions of the box that has the largest volume. Leave your answer(s) as fractions.



16. Find two positive numbers x and y satisfying x + 4y = 100, such that their product is a maximum.

17. Consider the function $f(x) = x^2 - x - 2$ on the interval [-1,2]. (*a.*) Verify that the hypotheses of Rolle's Theorem are satisfied. (*b.*) Find the number c in (-1,2) guaranteed by Rolle's Theorem.

18. Suppose f is continuous on [5,10], f(10) = 25, and $1 \le f'(x) \le 3$ for all x in the interval (5,10). What is the smallest possible value of f(5)?