Name: Key
M242: Calculus I (Fall 2018)
Midterm Exam, part I

Read and follow all instructions. You may not use any electronic devices.

Part I: Computations
Complete the following problems, showing enough work. Each problem is worth 5 points. Partial credit will be given when deserved.

1–3. Find the limits, provided they exist.

1. $\displaystyle\lim_{x\to 1^+} \frac{x-1}{\sqrt{x-1}} = \lim_{x\to 1^+} \sqrt{\frac{(x-1)^2}{x-1}} = \lim_{x\to 1^+} \sqrt{x-1} = \sqrt{0^+} = 0.$

2. $\displaystyle\lim_{x\to 2} \frac{x^2+5x-14}{x^2-3x+2} = \lim_{x\to 2} \frac{(x+7)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x+7}{x-1} = \frac{9}{1} = 9.$

3. $\displaystyle\lim_{x\to 0} \left(\frac{\sin(2x)}{x}\right)^3 = \left(\lim_{x\to 0} 2\frac{\sin(2x)}{2x}\right)^3 = 2^3\left(\lim_{u\to 0}\frac{\sin(u)}{u}\right) = 2^3\cdot 1 = 8$

4–7. Compute the derivatives of the functions. Show enough work.

4. $f(x) = (x+1)^2(2x-1)$ $= (x^2+2x+1)(2x-1) = 2x^3+4x^2+2x-x^2-2x-1$

$= 2x^3+3x^2-1$

$$\boxed{f'(x) = 6x^2+6x = 6x(x+1)}$$

5. $g(x) = \dfrac{x^2-2x}{\sqrt[3]{x}}$ $= \dfrac{x^2}{x^{1/3}} - 2\dfrac{x}{x^{1/3}} = x^{5/3} - 2x^{2/3}$

$g'(x) = \frac{5}{3}x^{2/3} - \frac{4}{3}x^{-1/3} = \boxed{\dfrac{5x-4}{3\sqrt[3]{x}} = g'(x)}$

6. $y = \sin\big(\tan(x^2)\big)$

$f = \sin(u)$ $\quad u = \tan(N)$ $\quad N = x^2$

$f' = \cos(u)$ $\quad u' = \sec^2(N)$ $\quad N' = 2x$

$$\boxed{y' = 2x\sec^2(x^2)\cos(\tan(x^2))}$$

7. $h(x) = \left(\dfrac{x}{x+1}\right)^2$ $= \dfrac{x^2}{x^2+2x+1}$

$h'(x) = \dfrac{(x^2+2x+1)(2x) - x^2(2x+2)}{(x+1)^4}$

$= \dfrac{\cancel{2x^3}+4x^2+2x-\cancel{2x^3}-2x^2}{(x+1)^4}$

$= \dfrac{2x\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}}$

$$\boxed{h'(x) = \frac{2x}{(x+1)^3}}$$

8. Find $\frac{dy}{dx}$ for the implicit function $x^2 + 6xy + y^2 = 0$.

$$\frac{d}{dx}\left[x^2+6xy+y^2=0\right]$$

$$2x + 6y + 6x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-6y}{6x+2y}}$$

9. Find an equation of the tangent line to the curve $y = \sec(x)$ at the point $\left(-\frac{\pi}{4}, \sqrt{2}\right)$.

$$\frac{dy}{dx} = \sec(x)\tan(x) \qquad \left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{4}} = \sec\left(\frac{-\pi}{4}\right)\tan\left(\frac{-\pi}{4}\right) = \sqrt{2}(-1) = -\sqrt{2}$$

$$y = y_0 + m(x-x_0)$$

$$= \sqrt{2} - \sqrt{2}(x+\pi/4)$$

$$\boxed{y = -\sqrt{2}x + \sqrt{2} - \frac{\pi}{2\sqrt{2}}}$$

10. Find all critical numbers of the function $y = x^3 - \frac{15}{2}x^2 + 18x - 100$.

$$\frac{dy}{dx} = 3x^2 - 15x + 18 = 3(x^2-5x+6) = 0$$

$$x^2 - 5x + 6 = (x-3)(x-2) = 0$$

$\boxed{x = 2, 3}$ Critical Numbers