

Midterm Exam, part II



WICHITA STATE
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Complete the following problems, showing enough work. Each problem is worth 10 points. Partial credit will be given when deserved.

1. You wish to prove that $\lim_{x \rightarrow 5} (10x - 7) = 43$. If you fix $\varepsilon > 0$, what should you set δ equal to in order to finish the proof? Show enough work to justify your answer.
2. Use the limit definition of the derivative to prove that $\frac{d}{dx} [x^5] = 5x^4$. You must use the limit definition to receive any credit. Show enough work.

Part II (b): Applications

Complete the following problems, showing enough work. Each problem is worth 10 points. Partial credit will be given when deserved.

3. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the interval. Then find the value(s) of c guaranteed by the theorem.

$$f(x) = x^3 - 6x^2 + 3x - 1, \quad 0 \leq x \leq 1$$

4. Consider the piecewise function

$$F(x) = \begin{cases} x^2 - 2x + 1 & x < 1 \\ k & x = 1 \\ \cos(x\pi) & x > 1 \end{cases}.$$

Is it possible for F to be continuous at $x = 1$? If so, what must k equal? Explain.

5. Use a linear approximation or differentials to approximate the value of $\sqrt{3.99}$. You may leave your answer as a fraction.
6. You wish to construct a rectangular box without a top such that the surface area of the box is 100 square inches. The base of the box must be a rectangle satisfying $\ell = 4w$. What are the dimensions of the box that maximize the volume? You may leave your answers as fractions.