PART I:
18 Oct '18

M242 - Calc I: Midterm Review

Brief Solutions

1. a.) $\lim_{x\to 0} \frac{x^2-2x+1}{x-1} = \lim_{x\to 0} \frac{(x-1)^2}{x-1} = \frac{(-1)^2}{-1} = \frac{1}{-1} = -1$

b.) $\lim_{x\to 1} \frac{x^2-2x+1}{x-1} = \lim_{x\to 1} \frac{(x-1)^2}{x-1} = \lim_{x\to 1} x-1 = 1-1 = 0$

c.) $\lim_{\theta\to\frac{\pi}{2}^-} \tan\theta = \tan\left(\pi/2^-\right) = +\infty$

d.) $\lim_{x\to 0} \frac{\sin(4x)}{x} = 4 \lim_{x\to 0} \frac{\sin(4x)}{4x} = 4 \lim_{u\to 0} \frac{\sin u}{u} = 4\cdot 1 = 4$

e.) $\lim_{y\to\frac{\pi}{4}} \ln(\tan(y)) = \ln\left(\lim_{y\to\pi/4} \tan(y)\right) = \ln(1) = 0$

f.) $\lim_{u\to\frac{\pi}{12}} \sin(u)\cos(u) = \lim_{u\to\pi/12} \frac{1}{2}\sin(2u) = \frac{1}{2}\sin\left(\lim_{u\to\pi/12} 2u\right) = \frac{1}{2}\sin(\pi/6) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}.$

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2. a.) $f(x) = 3x^2+9x-5$ $\qquad f'(x) = 6x+9$

b.) $g(x) = \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[5]{x}}$ $\qquad g(x) = \frac{\sqrt{x}}{\sqrt[5]{x}} - \frac{\sqrt[3]{x}}{\sqrt[5]{x}} = \frac{x^{1/2}}{x^{1/5}} - \frac{x^{1/3}}{x^{1/5}} = x^{3/10} - x^{2/15}$

So, $g'(x) = \frac{3}{10}x^{-7/10} - \frac{2}{15}x^{-13/15}$

c.) $h(x) = x^2\sin(x+1)$ $\qquad h'(x) = 2x\sin(x+1) + x^2\cos(x+1)$

d.) $v(\theta) = \tan\theta\cos\theta$ $\qquad v(\theta) = \frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta} = \sin\theta,\ \theta\neq\frac{\pi}{2}+n\pi$ $\qquad v'(\theta) = \cos\theta$

e.) $y = \tan(\cos(x))$ $\qquad \frac{dy}{dx} = -\sin x\sec^2(\cos x)$

f.) $K(p) = \sec^2(p)$ $\qquad K'(p) = 2\sec(p)\cdot\sec(p)\tan(p) = 2\sec^2(p)\tan(p).$

g.) $P(T) = \frac{nRT}{V}$ $\qquad \frac{dP}{dT} = \frac{nR}{V}$

h.) $q(x) = -\frac{x}{\sqrt{1+x^2}}$ $\qquad q'(x) = \frac{\sqrt{1+x^2}(-1) + x\frac{2x}{2\sqrt{1+x^2}}}{(\sqrt{1+x^2})^2} = \frac{-1-x^2+2x^2}{(\sqrt{1+x^2})^3} = \frac{x^2-1}{(\sqrt{1+x^2})^3}$

i.) $d(x) = \sqrt{1-(x-1)^2}$ $\qquad d'(x) = \frac{-\cancel{2}(x-1)}{\cancel{2}\sqrt{1-(x-1)^2}} = \frac{1-x}{\sqrt{1-(x-1)^2}}$

j.) $f(r) = (2r-1)^3(r+1)^2$

$$\frac{df}{dr} = 3(2r-1)^2\cdot 2\,(r+1)^2 + (2r-1)^3\cdot 2(r+1)$$

$$= (r+1)(2r-1)^2\left[6(r+1) + 2(2r-1)\right] = (r+1)(2r-1)^2(10r-4)$$

k.) $T(t) = \dfrac{2-\tan(t)}{\cos^2(t)}$

$$\frac{dT}{dt} = \frac{\cancel{\cos^2 t}\,(-\cancel{\sec^2 t}) + (2-\tan t)(2\cos t\sin t)}{\cos^4 t} = \frac{-1 + 4\cos t\sin t - 2\sin^2 t}{\cos^4 t}$$

l.) $u(y) = \csc(\sqrt{y^3 - 2y^2 + y})$

$$\frac{du}{dy} = \frac{-\csc\left(\sqrt{y^3-2y^2+y}\right)\cot\left(\sqrt{y^3-2y^2+y}\right)\left(3y^2-4y+1\right)}{2\sqrt{y^3-2y^2+y}}$$

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3. Find $\dfrac{dy}{dx}$: $x^2 + 6xy + y^2 = \sin(xy)$

$$\frac{d}{dx}\left[x^2 + 6xy + y^2 = \sin(xy)\right]$$

$$2x + 6y + 6x\frac{dy}{dx} + 2y\frac{dy}{dx} = \cos(xy)\left(y + x\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\cdot\left(6x + 2y - x\cos(xy)\right) = y\cos(xy) - 2x - 6y$$

so,

$$\boxed{\frac{dy}{dx} = \frac{y\cos(xy) - 2x - 6y}{6x + 2y - x\cos(xy)}}$$

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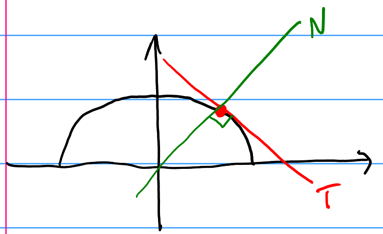
4. Find equations of the tangent and normal lines to the curve $y = \sqrt{1-x^2}$ at the point $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.



$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}} = \frac{-x}{y}$$

$$\left.\frac{dy}{dx}\right|_P = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

so, T: $y = \frac{\sqrt{2}}{2} + (-1)\left(x - \frac{\sqrt{2}}{2}\right) = -x + \sqrt{2}$

and N: $y = \frac{\sqrt{2}}{2} + 1\left(x - \frac{\sqrt{2}}{2}\right) = x$

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5. Find all points on the curve $y = x^2 - 2x + 1$ where the tangent line is parallel to the line $\underbrace{2x - 4y = 1}_{l}$.

l: $y = \frac{1}{4}(2x-1) = \frac{1}{2}x - \frac{1}{4}$ $\qquad m = \frac{1}{2}$

$\dfrac{dy}{dx} = 2x - 2 = \dfrac{1}{2} \Rightarrow x = \dfrac{5}{4}$ $\qquad y\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)^2 - 2\left(\frac{5}{4}\right) + 1 = \frac{25}{16} - \frac{40}{16} + \frac{16}{16} = \frac{1}{16}$

so the point is $P\left(5/4, \frac{1}{16}\right)$

PART II:
19 Oct '18

6. You wish to prove that $\lim_{x \to 2} 12x - 4 = 20$. If you fix $\varepsilon > 0$, what should you set δ equal to in order to finish the proof?

Suppose $0 < |x-2| < \delta$.

Then $|12x-4-20| = |12x-24| = 12|x-2| < 12\delta$

If $\delta = \frac{\varepsilon}{12}$, then

$$|12x-4-20| < 12\delta = 12 \frac{\varepsilon}{12} = \varepsilon$$ whenever $0<|x-2|<\delta$,

so put $\boxed{\delta = \frac{\varepsilon}{12}}$

7. Show that the equation $x^4 - 6x^2 = -5$ has a real root in the interval $(0, 2)$.

Intermediate Value Theorem (IVT)!

$x^4 - 6x^2 = -5 \Rightarrow x^4 - 6x^2 + 5 = 0$

$f(x) = x^4 - 6x^2 + 5$ is continuous on $[0,2]$ since it's a polynomial.

$f(0) = 5 > 0$

$f(2) = 2^4 - 6 \cdot 2^2 + 5 = 16 - 24 + 5 = -3 < 0$

The IVT then states that f must have at least one real zero in the interval $(0,2)$.

8. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the interval. Then find the values of c that are guaranteed by the theorem.

$$f(x) = x^3 - 3x^2 + x - 1, \quad 0 \le x \le 2$$

MVT: $f(x) = x^3 - 3x^2 + x - 1$ is continuous and differentiable on $\mathbb{R}$ since it is a polynomial.

$$m_s = \frac{f(2) - f(0)}{2 - 0} = \frac{2^3 - 3 \cdot 2^2 + 2 - 1 - (-1)}{2} = \frac{8 - 12 + 2 + 1}{2} = \frac{-1}{2}$$

$f'(x) = 3x^2 - 6x + 1$

Now solve $3x^2 - 6x + 1 = \frac{-1}{2}$ for x.

$$x^2 - 2x + 1 = -\frac{1}{2} + 1$$

$$(x-1)^2 = \frac{1}{2}$$

$$x = 1 \pm \frac{\sqrt{2}}{2}$$

only $x = 1 + \frac{\sqrt{2}}{2}$ is in $(0,2)$, so $\boxed{c = 1 + \frac{\sqrt{2}}{2}}$

9. Use the limit definition of derivative to compute $\frac{dy}{dx}$ for the given functions. You must use the limit definition to receive credit on the exam.

a.) $y = \sqrt{x}$

$$f'(a) = \lim_{x\to a} \frac{(\sqrt{x}-\sqrt{a})}{(x-a)}\frac{(\sqrt{x}+\sqrt{a})}{(\sqrt{x}+\sqrt{a})} = \lim_{x\to a} \frac{\cancel{x-a}\ 1}{(\cancel{x-a})(\sqrt{x}+\sqrt{a})} = \lim_{x\to a}\frac{1}{\sqrt{x}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

so $\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$

b.) $y = \frac{1}{x}$

$$f'(a) = \lim_{x\to a}\frac{\frac{a}{a}\frac{1}{x} - \frac{1}{a}\frac{x}{x}}{x-a} = \lim_{x\to a}\frac{-(\cancel{x-a})\ 1}{xa(\cancel{x-a})} = \lim_{x\to a}\frac{-1}{xa} = \frac{-1}{a^2}$$

So, $\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{-1}{x^2}$

c.) $y = x^5$

Division!

$$f'(a) = \lim_{x\to a}\frac{x^5-a^5}{x-a} = \lim_{x\to a}\left(x^4+ax^3+a^2x^2+a^3x+a^4\right) = a^4+a^4+a^4+a^4+a^4 = 5a^4$$

so, $\frac{d}{dx}\left[x^5\right] = 5x^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
$\boxed{1\ 5\ 10\ 10\ 5\ 1}$

OR

multiplication!

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^5 - x^5}{h} = \lim_{h\to 0}\frac{(\cancel{x^5}+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5)-\cancel{x^5}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(5x^4+10x^3h+10x^2h^2+5xh^3+h^4)}{\cancel{h}}$$

$$= 5x^4 + 0 + 0 + 0 + 0$$

$$= 5x^4$$

10. Consider the piecewise function

$$F(x) = \begin{cases} x^2+2x-1 & x<0 \\ k & x=0 \\ \cos(x-\pi) & x>0 \end{cases}.$$

Is it possible for F to be continuous at $x = 0$? If so, what must k be equal to? Explain.

F is continuous at $x=0$ iff $\lim_{x\to 0} F(x) = F(0) = k$.

Question: Does $\lim_{x\to 0} F(x)$ exist?

$$\left.\begin{aligned}\lim_{x\to 0^-} F(x) &= \lim_{x\to 0^-}(x^2+2x-1) = -1 \\ \lim_{x\to 0^+} F(x) &= \lim_{x\to 0^+}\cos(x-\pi) = \cos(-\pi) = -1\end{aligned}\right\}$$ These agree, so $\lim_{x\to 0} F(x) = -1$.

Thus, in order for F to be continuous at $x=0$, $F(0)$ must equal -1.

So put $k=-1$.

11. Find the Taylor polynomial T_5 for the function $f(x) = \sin(x)$ at $x = 0$.

For any function f w/ k-derivatives, $T_{k,x_0}(x) = \sum_{n=0}^{k} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$.

So $T_5(x) = T_{5,0}(x) = \underbrace{f(0)}_{0} + \underbrace{f'(0)}_{1} \cdot x + \frac{1}{2}\underbrace{f''(0)}_{0} \cdot x^2 + \frac{1}{6}\underbrace{f'''(0)}_{-1} \cdot x^3 + \frac{1}{24}\underbrace{f^{(4)}(0)}_{0} \cdot x^4 + \frac{1}{120}\underbrace{f^{(5)}(0)}_{1} \cdot x^5$

Need to compute the $f^{(n)}(0)$'s:

n	$f^{(n)}(x)$	$\xrightarrow{x=0}$	$f^{(n)}(0)$
0	$\sin x$		0
1	$\cos x$		1
2	$-\sin x$		0
3	$-\cos x$		-1
4	$\sin x$		0
5	$\cos x$		1

so, $\boxed{T_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5}$

12. Find the Taylor polynomial T_3 for the function $f(x) = \sin(x)$ at $x = \frac{\pi}{4}$.

Same scenario, but now $x_0 = \pi/4$, so $T_3(x) = \underline{f(\pi/4)} + \underline{f'(\pi/4)}(x-\pi/4) + \frac{1}{2}\underline{f''(\pi/4)}(x-\pi/4)^2 + \frac{1}{6}\underline{f'''(\pi/4)}(x-\pi/4)^3$

n	$f^{(n)}(x)$	$\xrightarrow{x=\pi/4}$	$f^{(n)}(\pi/4)$
0	$\sin x$		$\sqrt{2}/2$
1	$\cos x$		$\sqrt{2}/2$
2	$-\sin x$		$-\sqrt{2}/2$
3	$-\cos x$		$-\sqrt{2}/2$

so, $\boxed{T_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x-\pi/4) - \frac{\sqrt{2}}{4}(x-\pi/4)^2 - \frac{\sqrt{2}}{12}(x-\pi/4)^3}$

13. Use a linear approximation or differentials to approximate the values of $\sqrt{25.1}$ and $\sqrt{24.99}$.

$f(x) = \sqrt{x}$ $\quad x_0 = 25$

$L_{f,x_0}(x) = f(x_0) + \underbrace{f'(x_0)(x-x_0)}_{dy}$ $\qquad f(x_0+dx) \approx f(x_0) + dy = L_{f,x_0}(x_0+dx)$

$f'(x) = \frac{1}{2\sqrt{x}}$ so $f'(25) = \frac{1}{10}$ and $dy = \frac{1}{10}dx$, $f(25) = 5$

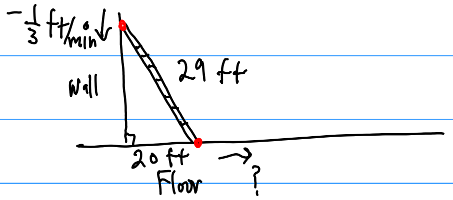
so, $L(x) = 5 + \frac{1}{10}(x-25)$,

and

$\sqrt{25.1} \approx 5 + \frac{1}{10}(25.1-25) = 5 + \frac{1}{10}\left(\frac{1}{10}\right) = 5 + \frac{1}{100} = \frac{501}{100} = 5.01$

$\sqrt{24.99} \approx 5 + \frac{1}{10}(-0.01) = 5 + \frac{1}{10}\left(\frac{-1}{100}\right) = 5 - \frac{1}{1000} = 4.999$

14. A 29 ft ladder is sliding down a wall at a constant rate of 4 inches per minute. Find the rate at which the base of the ladder is sliding away from wall when the base of the ladder is 20 ft from the base of the wall.



$y^2+x^2=29^2 \xrightarrow{\text{Implicit Der.}} \cancel{2}y\frac{dy}{dt}+\cancel{2}x\frac{dx}{dt}=0$

$y^2+20^2=29^2$

$21\left(-\frac{1}{3}\right)+20\frac{dx}{dt}=0$

$y=\sqrt{29^2-20^2}=21$ ft.

So $\frac{dx}{dt}=\frac{7}{20}$ ft/min

$=\boxed{0.35 \text{ ft/min}}$

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15. Find all relevant information about the graphs of the functions, then sketch the graph.

a.) $y=x^3-3x^2-9x+27 = x^2(x-3)-9(x-3)=(x^2-9)(x-3)=(x+3)(x-3)^2$

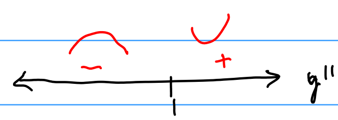
y-int: $(0,27)$

x-int: $(-3,0)$ and $(3,0)$ cubic function, positive LC,

CN: $y'=3x^2-6x-9=3(x^2-2x-3)=3(x-3)(x+1)=0 \Rightarrow x=3,\ x=-1$

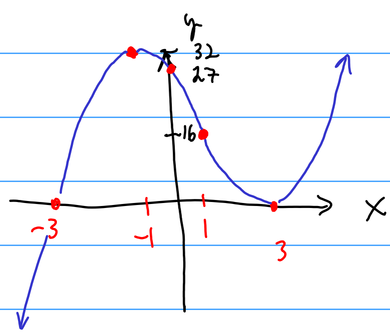
CP: $(3,0)$ and $(-1,32)$

SDT: $y''=6x-6=6(x-1) \Rightarrow x=1$



IP: $(1,16)$

Sketch:



"$y=x^3-3x^2-9x+27$"

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b.) $y=\dfrac{\sqrt{x^2-1}}{x-1}$

Domain: $x^2-1\geq 0 \Rightarrow x^2\geq 1 \Rightarrow x\geq 1$ and $x\leq -1$

$x-1\neq 0 \Rightarrow x\neq 1$

Together, domain: $(-\infty,-1]\cup(1,\infty)$

y-int: DNE! $x=0$ not in domain.

x-int: $x=-1$ only ($x=1$ not in domain!) : $(-1,0)$

end/asymp. behavior: $\lim\limits_{x\to 1^+}\frac{\sqrt{x^2-1}}{x-1}=\lim\limits_{x\to 1^+}\sqrt{\frac{x^2-1}{(x-1)^2}}=\lim\limits_{x\to 1^+}\sqrt{\frac{(x-1)(x+1)}{(x-1)^2}}=\lim\limits_{x\to 1^+}\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{2}{0^+}}=+\infty$, VA!

$\lim\limits_{x\to+\infty}\frac{\sqrt{x^2-1}}{x-1}=\lim\limits_{x\to+\infty}\sqrt{\frac{x^2-1}{(x-1)^2}}=\lim\limits_{x\to+\infty}\sqrt{\frac{1x^2-1}{1x^2-2x+1}}\to\sqrt{1}=1$ HA

$\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-1}}{x-1}=\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-1}}{-\sqrt{(x-1)^2}}=\lim\limits_{x\to-\infty}-\sqrt{\frac{x^2-1}{x^2-2x+1}}=-\sqrt{1}=-1$ HA

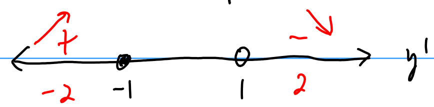
CN: $y' = \dfrac{(x-1)\frac{x}{\sqrt{x^2-1}} - \sqrt{x^2-1}\,(1)}{(x-1)^2} = \dfrac{\cancel{x^2} - x - \cancel{x^2} + 1}{(x-1)^2\sqrt{x^2-1}} = \dfrac{-\cancel{(x-1)}\;1}{(x-1)^{\cancel{2}}\sqrt{x^2-1}} = \dfrac{-1}{(x-1)\sqrt{x^2-1}}$

top: None

bott: $x = -1$ ($x=1$ not in domain.)

$x=-1$ is an endpoint and corresponds to a vertical tan. line.

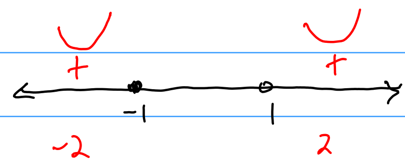
FDT:



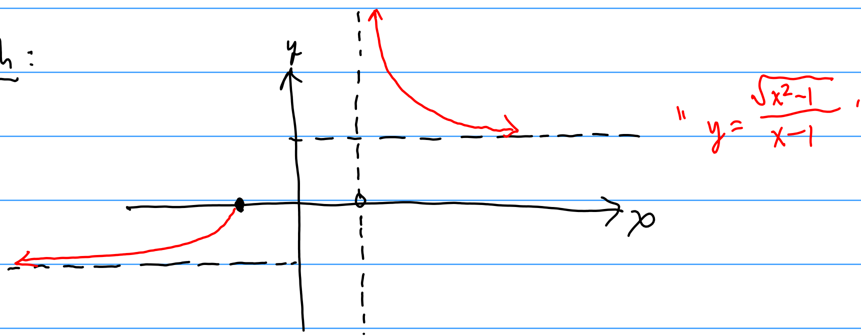
SDT: $y'' = \dfrac{\sqrt{x^2-1} + (x-1)\frac{x}{\sqrt{x^2-1}}}{(x-1)^2(x^2-1)} = \dfrac{x^2-1+x^2-x}{(x-1)^2\sqrt{x^2-1}^{\,3}} = \dfrac{2x^2-x-1}{(x-1)^2\sqrt{x^2-1}^{\,3}} = \dfrac{(2x+1)\cancel{(x-1)}}{(x-1)^{\cancel{2}}\sqrt{x^2-1}^{\,3}} = \dfrac{2x+1}{(x-1)\sqrt{x^2-1}^{\,3}}$

$2x^2-x-1 = 2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{2} - \frac{1}{16}\right) = 0 \Rightarrow (x-1/4)^2 = \frac{1}{2} + \frac{1}{16}$

$x = 1/4 \pm 3/4 = -1/2,\ 1$ Neither in domain.



Sketch:



c.) $y = \dfrac{x^3}{x^2+3x+2} = \dfrac{x^3}{(x+2)(x+1)} = x - 3 + \dfrac{7x+6}{(x+2)(x+1)} = x-3+\dfrac{A}{x+2}+\dfrac{B}{x+1} = x-3+\dfrac{8}{x+2}-\dfrac{1}{x+1}$

$$\begin{array}{r l}
 & x \quad -3 \\
x^2+3x+2 \,\big) & x^3 \\
 & x^3+3x^2+2x \\
 & -3x^2-2x \\
 & -(-3x^2-9x-6) \\
 & 7x+6
\end{array}$$

$7x+6 = A(x+1) + B(x+2)$

$x=-1$: $-1 = B$

$x=-2$: $-8 = -A \Rightarrow A = 8$

Domain: $x \neq -1, -2$

x-int: $(0,0)$

y-int: $(0,0)$

end/asymptote behavior: $\lim_{x\to -1^-}\left(-\frac{1}{x+1}\right) = -\frac{1}{0^-} = +\infty$ $\quad \lim_{x\to -2^-}\left(\frac{8}{x+2}\right) = \frac{8}{0^-} = -\infty$

$\lim_{x\to -1^+}\left(\frac{-1}{x+1}\right) = -\frac{1}{0^+} = -\infty$ $\quad \lim_{x\to -2^+}\left(\frac{8}{x+2}\right) = \frac{8}{0^+} = +\infty$

As $x \to \pm\infty$, $y \to x-3$ (slant asymptote)

FDT: $y' = 1 - \dfrac{8}{(x+2)^2} + \dfrac{1}{(x+1)^2} = \dfrac{(x+1)^2(x+2)^2 - 8(x+1)^2 + (x+2)^2}{((x+1)(x+2))^2} = \dfrac{(x^2+3x+2)^2 - 8(x^2+2x+1) + (x^2+4x+4)}{(x^2+3x+2)^2}$

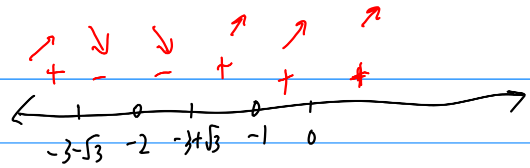
$= \dfrac{x^4+3x^3+2x^2+3x^3+9x^2+6x+2x^2+6x+4-8x^2-16x-8+x^2+4x+4}{(x^2+3x+2)^2}$

$$= \frac{x^4+6x^3+6x^2}{(x^2+3x+2)^2} = \frac{x^2(x^2+6x+6)}{(x^2+3x+2)^2}$$

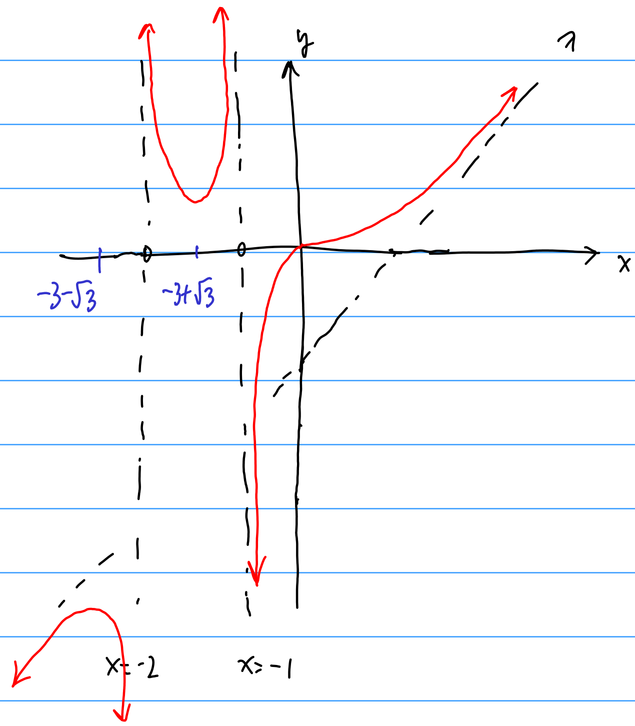
CN: $x^2=0 \Rightarrow x=0$

$x^2+6x+9 = -6+9$

$(x+3)^2 = 3 \Rightarrow x = -3 \pm \sqrt{3}$



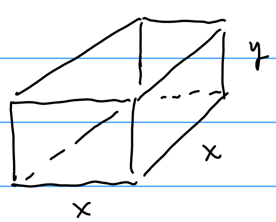
Sketch:



↗ $x-3$

" $y = \frac{x^3}{x^2+3x+2}$ "

16. You wish to construct a rectangular box without a top such that the surface area of the box is 10 square inches. What are the dimensions of the box that maximize the volume?



Not enough info. Also assume the base of the box is a square.

$$A = x^2 + 4xy = 10 \Rightarrow y = \frac{10-x^2}{4x}$$

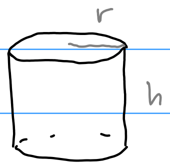
$$V = x^2y = x^2\left(\frac{10-x^2}{4x}\right) = \frac{1}{4}x(10-x^2) = \frac{10}{4}x - \frac{1}{4}x^3$$

$$\frac{dV}{dx} = \frac{10}{4} - \frac{3}{4}x^2 = 0 \Rightarrow x^2 = \frac{10}{3}, \quad \boxed{x = \sqrt{\frac{10}{3}}}$$

$$y = \frac{10-\left(\frac{10}{3}\right)}{4\sqrt{10/3}} = \frac{20}{3\cdot 4\cdot\sqrt{10}}\sqrt{3} = \frac{2\sqrt{10}}{4\sqrt{3}} = \boxed{\frac{1}{2}\sqrt{\frac{10}{3}} = y}$$

So the dimensions of the box are: $\sqrt{\frac{10}{3}} \times \sqrt{\frac{10}{3}} \times \frac{1}{2}\sqrt{\frac{10}{3}}$

17. You wish to construct an aluminum bucket in the shape of a right circular cylinder with a volume of 1 L. If the bucket does not have a top, find the dimensions of the cylinder that minimize the surface area.



$$V = \pi r^2 h = 1000 \text{ cm}^3, \quad \text{so} \quad h = \frac{1000}{\pi r^2}$$

$$A = \pi r^2 + 2\pi r h = \pi r^2 + 2\cancel{\pi r}\frac{1000}{\cancel{\pi} r^{\cancel{2}}} = \pi r^2 + \frac{2000}{r}$$

$$\frac{dA}{dr} = 2\pi r - \frac{2000}{r^2} = 0 \Rightarrow r^3 = \frac{2000}{2\pi} \rightarrow \boxed{r = \sqrt[3]{\frac{1000}{\pi}} \text{ cm}}$$

$$h = \frac{1000}{\pi} \cdot \left(\frac{1000}{\pi}\right)^{-2/3} = \boxed{\sqrt[3]{\frac{1000}{\pi}} \text{ cm} = h}$$

18. Suppose f is continous on $[1,5]$, $f(1) = 2$, and $3 \le f'(x) \le 5$ for all x in the interval $(1,5)$. What are the largest and smallest possible values of $f(5)$?

MVT: $f'(c) = \dfrac{f(5) - f(2)}{5-1}$ for some c in $(1,5)$.

$f(5) = f(2) + 4 \cdot f'(c) = 2 + 4f'(c)$.

and $3 \le f'(c) \le 5$, so

$2 + 4 \cdot 3 \le f(5) \le 2 + 4 \cdot 5$

or $\boxed{14 \le f(5) \le 22}$