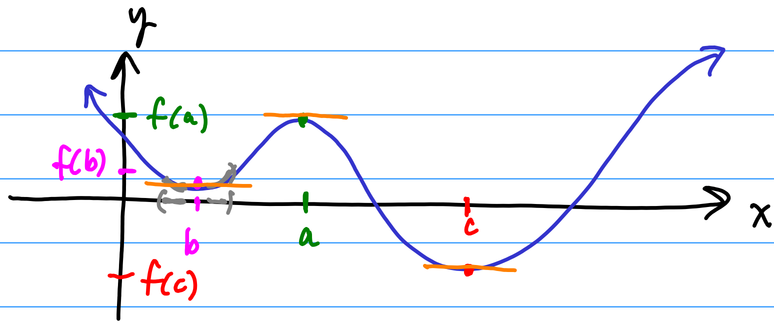
M242 9/20/18

§3.1 – Maxima and Minima

$y = f(x)$



$(a, f(a))$ is a local or relative maximum of the function if x is near a ($|x-a|<\varepsilon$), then $f(x) \le f(a)$.

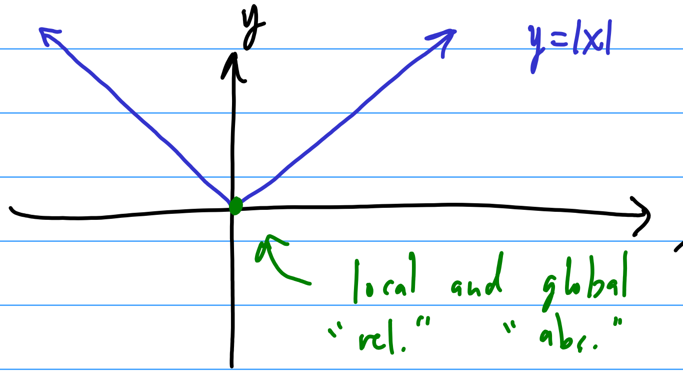
$(b, f(b))$ is a local minimum: if x is near b ($|x-b|<\varepsilon$), then $f(x) \ge f(b)$.

$(c, f(c))$ is also a local minimum

$(c, f(c))$ is a global (or absolute) minimum because it's the lowest point on the graph: $f(x) \ge f(c)$ for all x in domain of f.

Calculus Fact: If $f'(x)$ exists at a local maximum or minimum, then $f'(x)=0$ at these points. (horizontal tan. line)

Ex. $f(x)=|x|$ Find the max/min of f.



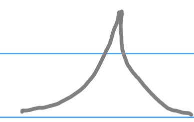
No local max.

local and global minimum: $(0,0)$
"rel." "abs."

BUT $f'(0)$ = undef.

Defn. A critical number of a function f is a number c in the domain of f such that

$$\begin{cases} f'(c) = 0 \\ \text{or } f'(c) = \text{undef.} \end{cases}$$



The corresponding critical point $(c, f(c))$ lies on the graph of f.

Fermat's Theorem. If f has a local extreme point at $(c, f(c))$, then $(c, f(c))$ must be a critical point of f.

Ex. $f(x) = x^3$

$f'(x) = 3x^2$

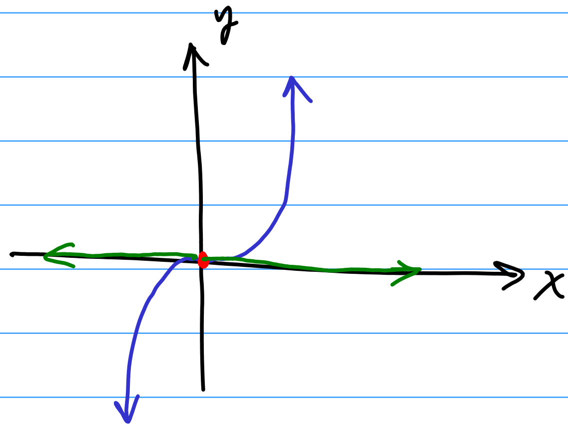
$3x^2 = 0$

$x^2 = 0$

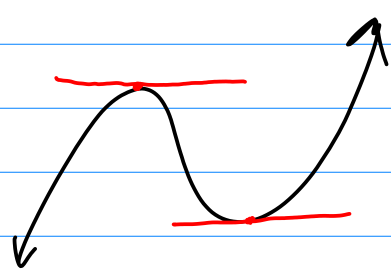
$x = 0$

$(0,0)$ is a C.P.

BUT not a max/min



e.g.,



Ex. $f(x) = 3x^4 - 16x^3 + 18x^2$ Find the CNs of f.

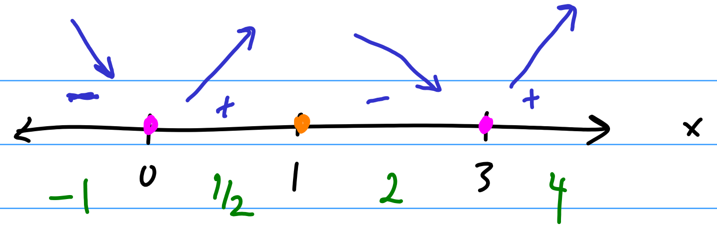
$$f'(x) = 12x^3 - 48x^2 + 36x = 12x(x^2 - 4x + 3) = 0$$

$$= \boxed{12x(x-3)(x-1)} = 0$$

$\boxed{x = 0 \quad x = 3 \quad x = 1}$ C.N.

To check whether these are peaks or valleys we can make a sign chart:

$f'(x) = \boxed{12}x(x-3)(x-1)$



$f'(-1) = (-)(-)(-) = -$

$f'(1/2) = + \; - \; - = +$

$f'(2) = + \; - \; + = -$

$f'(4) = + \; + \; + = +$

Local minima: $x=0$, $x=3$

Local maximum: $x=1$

Now plug back into f to find the heights:

$f(x) = 3x^4 - 16x^3 + 18x^2$

$f(3) = 243 - 16(27) + 18(9) = 243 + 162 - 432$
$= 405 - 432 = -27$

local min: $(0,0)$ and $(3,-27)$

local max: $(1,5)$

Since f is a deg. 4 polynomial w/ positive leading coeff., the lowest min is the absolute min:

$(3,-27)$ is global minimum

Ex. $y = mx + b$ Find max/min's.

$y' = \underbrace{m = 0}$ Either no CNs or all CNs.