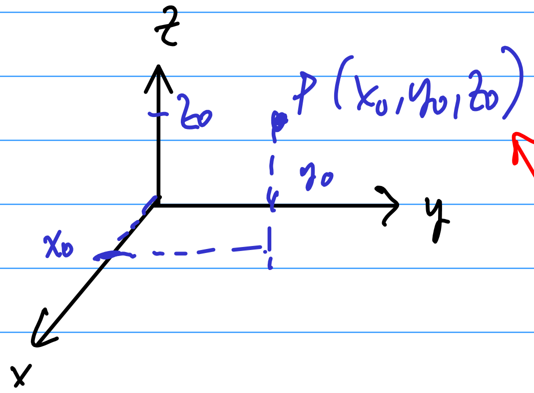
Math 243 — Calc II

8/20

§12.1 - 3D Cartesian Coordinates



Rectangular or Cartesian coordinates

Defn. Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.
The distance between P and Q is the real number

$$d(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Ex. $P(1,2,3)$ $\quad Q(-1,0,2)$

Find $d(P,Q) = \sqrt{(1-(-1))^2 + (2-0)^2 + (3-2)^2}$

$$= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4+4+1} = \sqrt{9} = 3.$$

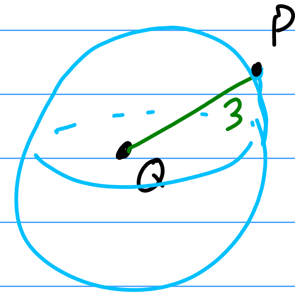
Ex. Describe the set of points (x,y,z) satisfying

$$\sqrt{(x-1)^2 + (y+2)^2 + z^2} = \sqrt{9}$$

$P(x,y,z)$ $\quad Q(1,-2,0)$

$d(P,Q) = 3$

A. Sphere centered at $(1,-2,0)$ w/ radius 3.

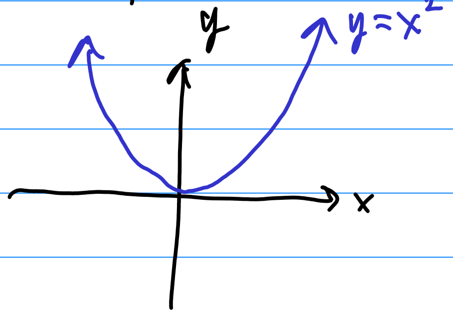


Def'n. A **sphere** centered at $C(h,k,\ell)$ w/ radius ρ (rho), $\rho > 0$, is given by

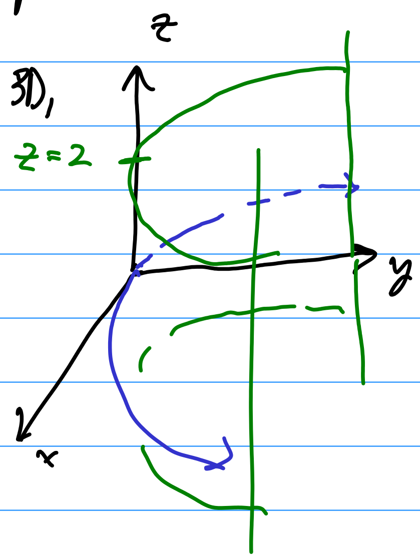
$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = \rho^2 \qquad (*)$$

Ex. $y = x^2$ Describe the graph.

In 2D,



In 3D,



Notation:
- $\mathbb{R}$ = all real numbers
- $\mathbb{R}^2$ = xy-plane or 2D space
- $\mathbb{R}^3$ = xyz-space or 3D space