M243 – Calc III 8/21

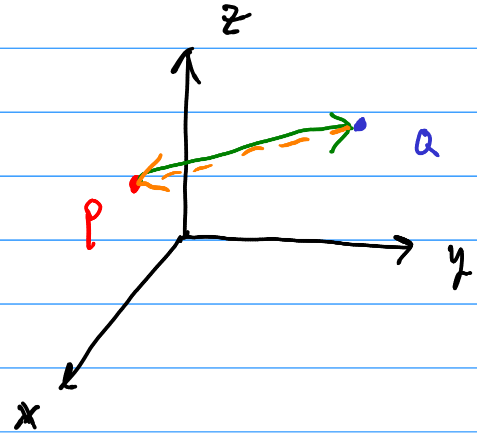
§12.2 – Vectors

Def'n. A vector is a quantity that has both magnitude and direction.

Examples. velocity, acceleration, gravity, Forces

speed is not, distance is not } scalars

For us, a vector is an arrow (directed line segment) in $\mathbb{R}^2$ or $\mathbb{R}^3$.



$\vec{v} = \overrightarrow{PQ}$

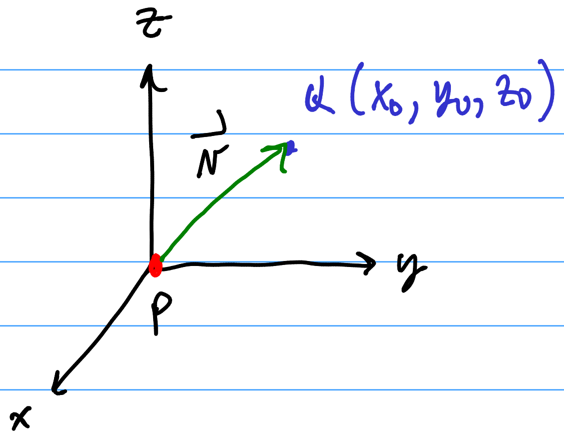
$\overrightarrow{QP} \neq \overrightarrow{PQ}$

Def'n. Two vectors are said to be equivalent iff they have the same magnitude and direction.

(*) We can translate our vectors in space w/out changing them!

Def'n. A vector is said to be in standard position if its initial point is the origin.

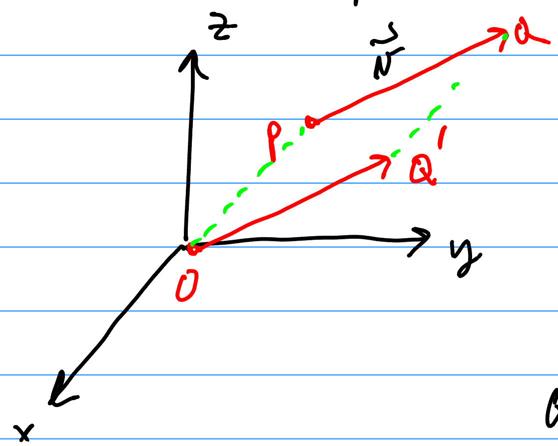
Ex.



The vector $\vec{v}$ in standard position ($P=0$) has component form

$\vec{v} = \langle x_0, y_0, z_0 \rangle$

Ex. $\overrightarrow{PQ}$ not in standard pos.



$P(x_0, y_0, z_0)$
$Q(x_1, y_1, z_1)$
what are the coord's of Q'?

$Q' = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$

$\vec{v} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

"terminal minus initial"

Ex. $P(0, 3, -1)$ $Q(5, 2, 3)$
Find comp. form of $\overrightarrow{PQ}$.

$\overrightarrow{PQ} =$ "$Q - P$" $= \langle 5-0, 2-3, 3-(-1) \rangle$
$\vec{v} = \langle 5, -1, 4 \rangle$

Defn. given a vector $\vec{v}$ in comp form, $\vec{v} = \langle x_0, y_0, z_0 \rangle$, its magnitude is

$$\|\vec{v}\| = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

Ex. Find the magnitude of $\overrightarrow{PQ}$ where

$$\left.\begin{matrix} P(5,1,-3) \\ Q(3,-1,-2) \end{matrix}\right\} \quad \vec{v} = \langle -2,-2,1 \rangle$$

$$d(P,Q) = \|\vec{v}\| = \|\overrightarrow{PQ}\|$$

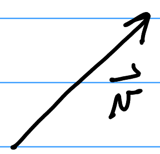
so,

$$\|\vec{v}\| = \sqrt{(-2)^2+(-2)^2+1^2} = \sqrt{9} = 3$$

Algebra:

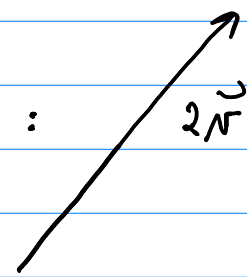
Scalar Mult. $\alpha \in \mathbb{R}$, $\vec{v}$ a vector

we can multiply $\alpha\vec{v}$ to make a new vector.



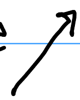
$\alpha = 2$.

$2\vec{v}$:



Same direction
α-times mag.

$\frac{1}{2}\vec{v}$

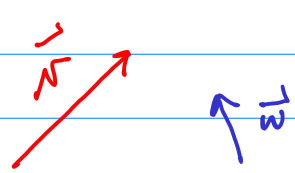


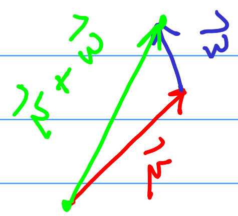
$0\vec{v}$.

Component-wise, $\vec{v} = \langle x_0, y_0, z_0 \rangle$

$$\alpha\vec{v} = \alpha\langle x_0, y_0, z_0 \rangle = \langle \alpha x_0, \alpha y_0, \alpha z_0 \rangle$$

Vector Addition: $\vec{v}, \vec{w}$ we can add these to create a new vector.





"tip-to-tail"

In comp. form, $\vec{v} = \langle x_0, y_0, z_0 \rangle$
$\vec{w} = \langle x_1, y_1, z_1 \rangle$

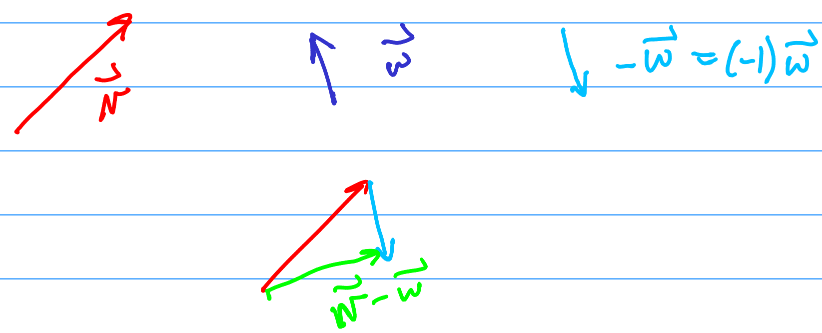
$$\vec{v} + \vec{w} = \langle x_0 + x_1, y_0 + y_1, z_0 + z_1 \rangle$$

"component by component"

<u>RE.</u> $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

Draw the picture.

<u>Ex.</u> $\vec{v} - \vec{w} = \vec{v} + (-1)\vec{w}$ <u>Subtraction</u>



<u>Ex.</u>