M243

§12.2

Properties. Let $\bar{u}, \bar{v}, \bar{w}$ be vectors in $\mathbb{R}^n$, and let a, b be real numbers (scalars).

1. $\bar{u} + \vec{v} = \vec{v} + \bar{u}$
2. $\bar{v} + \bar{0} = \bar{v}$
3. $\bar{u} + (\bar{v} + \bar{w}) = (\bar{u} + \bar{v}) + \bar{w} = \bar{u} + \bar{v} + \bar{w}$
4. $\bar{u} + (-\bar{u}) = \bar{0}$
5. $a(\bar{u} + \bar{v}) = a\bar{u} + a\bar{v}$
6. $(a+b)\bar{u} = a\bar{u} + b\bar{u}$
7. $(ab)\bar{u} = a(b\bar{u}) = b(a\bar{u})$
8. $1\bar{u} = \bar{u}$

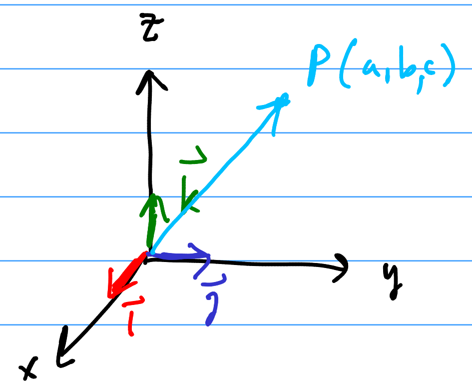
Standard Unit Vectors

In $\mathbb{R}^3$,

$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$



$\vec{v} = \langle a, b, c \rangle$

$$\vec{v} = \langle a,0,0 \rangle + \langle 0,b,0 \rangle + \langle 0,0,c \rangle$$

$$= a\underline{\langle 1,0,0 \rangle} + b\underline{\langle 0,1,0 \rangle} + c\underline{\langle 0,0,1 \rangle} = a\vec{i} + b\vec{j} + c\vec{k} = \vec{v}$$

Ex. $\bar{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ $\quad\quad \vec{v} = 3\vec{a} + 2\vec{b}$

$\bar{b} = 4\vec{i} + 7\vec{k}$ $\quad\quad$ Write $\vec{v}$ in i, j, k - form.

$$\vec{v} = 3\vec{a} + 2\vec{b} = 3(\vec{i} + 2\vec{j} - 3\vec{k}) + 2(4\vec{i} + 7\vec{k})$$

$\vec{v} = 3\vec{i} + 6\vec{j} - 9\vec{k} + 8\vec{i} + 14\vec{k}$

so, $\boxed{\vec{v} = 11\vec{i} + 6\vec{j} + 5\vec{k}} \rightarrow \vec{v} = \langle 11, 6, 5 \rangle$

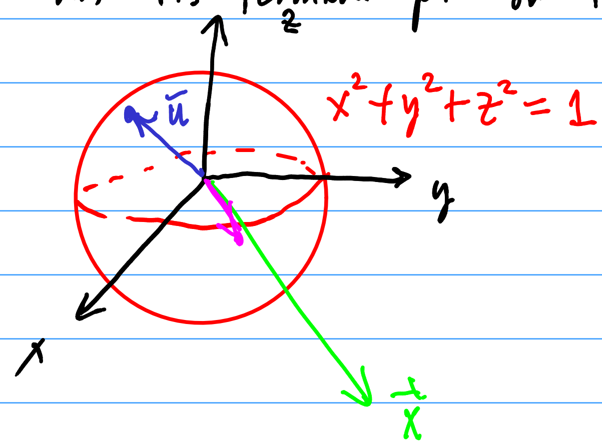
<u>Def'n</u>. A <u>unit</u> <u>vector</u> is any vector whose length is 1.

$$\bar{u} = \langle u_1, u_2, u_3 \rangle \qquad \|\bar{u}\| = 1$$

$$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2} = 1$$

$$u_1^2 + u_2^2 + u_3^2 = 1$$

- Every unit vector has its terminal pt on the unit sphere:



<u>Ex</u>. $\bar{x} = 2\vec{i} - \vec{j} - 2\vec{k}$ Is this a unit vector? <u>No</u>.
If not, find one w/ the same dir.

$$\|\bar{x}\| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

The unit vector will be $\bar{u} = \dfrac{\bar{x}}{\|\bar{x}\|} = \dfrac{1}{\|\bar{x}\|}\,\bar{x}$

$$\bar{u} = \frac{1}{3}\langle 2, -1, -2 \rangle = \langle \tfrac{2}{3}, \tfrac{-1}{3}, \tfrac{-2}{3} \rangle$$

or $\bar{u} = \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$

§12.3 – Dot Product

Defn. Let $\vec{v}, \vec{w} \in \mathbb{R}^3$. The dot product (scalar product) of $\vec{v}$ and $\vec{w}$ is the real number determined by

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

where $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$.

"multiply component-by-component, then add 'em up!"

Ex. $\vec{v} = \langle 1, 2, 3 \rangle$, $\vec{w} = \langle -2, 1, 6 \rangle$

$$\vec{v} \cdot \vec{w} = 1(-2) + 2(1) + 3(6) = -2 + 2 + 18 = 18.$$

Property: $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ for all vectors $\vec{v}, \vec{w}$.

Ex. $\vec{v} = \langle v_1, v_2, v_3 \rangle$ $\quad \vec{0} = \langle 0, 0, 0 \rangle$

$$\vec{v} \cdot \vec{0} = v_1(0) + v_2(0) + v_3(0) = 0 + 0 + 0 = 0$$

Ex. $\vec{v} = \langle 1, 3, 4 \rangle$ $\quad \vec{w} = \langle 0, 4, -3 \rangle$

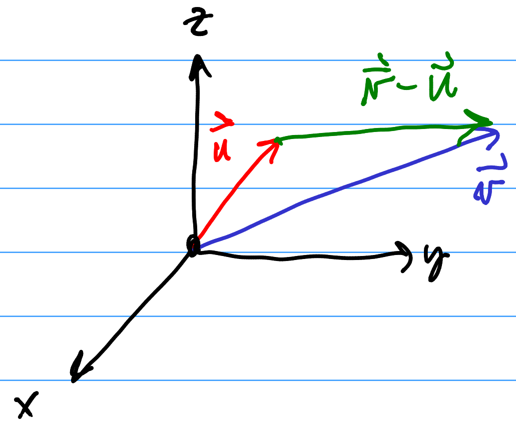
$$\vec{v} \cdot \vec{w} = 1(0) + 3(4) + 4(-3) = 0 + 12 - 12 = 0$$

Ex. Let $\vec{v} = \langle v_1, v_2, v_3 \rangle$.

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{v_1 v_1 + v_2 v_2 + v_3 v_3} = \sqrt{\vec{v} \cdot \vec{v}}$$

So, $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$ and $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$

Ex.



$d(\vec{u},\vec{v}) = \|\vec{v}-\vec{u}\| = \|\vec{u}-\vec{v}\|$