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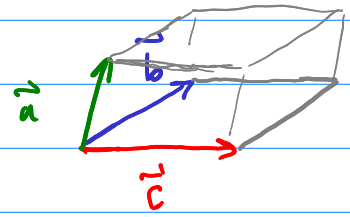
§12.4, cont'd.

5. $\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$ Scalar Triple Product



parallelepiped

The scalar triple product $\vec{a}\cdot(\vec{b}\times\vec{c})$ is the volume of the parallelepiped determined by $\vec{a}, \vec{b}, \vec{c}$.

Ex. P(-2, 1, 0)
Q(2, 3, 2)
R(1, 4, -1)
S(3, 6, 1)

Determine the volume of the p-piped w/ edges $\vec{PQ}, \vec{PR}, \vec{PS}$.

$\vec{PQ} = \langle 4, 2, 2\rangle$
$\vec{PR} = \langle 3, 3, -1\rangle$
$\vec{PS} = \langle 5, 5, 1\rangle$

Volume = 16

$$\vec{PQ}\cdot(\vec{PR}\times\vec{PS}) = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 3 & -1 \\ 5 & 5 & 1 \end{vmatrix} = 4(3+5) - 2(3+5) + 2(15-15)$$

$$= 2(8) = 16.$$

Ex. $\vec{a} = \langle 1, 4, -7\rangle$
$\vec{b} = \langle 2, -1, 4\rangle$
$\vec{c} = \langle 0, -9, 18\rangle$

Are these vectors coplanar?

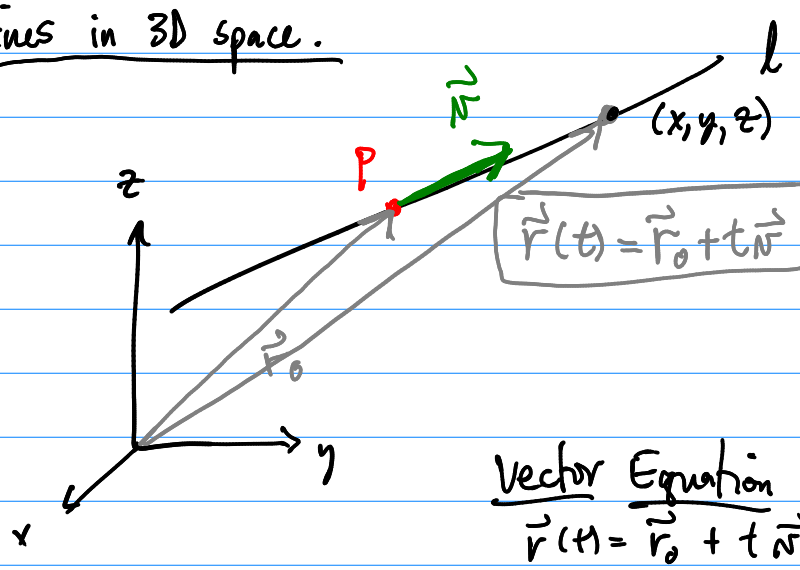
$\vec{a}\cdot(\vec{b}\times\vec{c}) \stackrel{?}{\neq} 0$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 1(-18+36) - 4(36-0) - 7(-18-0)$$

$$= 18 - 4(36) + 7\cdot 18 = 8\cdot 18 - 8\cdot 18 = 0$$

§12.5 Equations of lines and planes in space.

Lines in 3D space.



$P(x_0, y_0, z_0)$

$\vec{v} = \langle a, b, c \rangle$

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$ pos. vector for P

Vector Equation of the line l:

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

$$\vec{r}(t) = \langle x, y, z \rangle = \langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle \quad \circledast$$

Parametric Equations:

$$\begin{cases} x(t) = x_0 + at \\ y(t) = y_0 + bt \\ z(t) = z_0 + ct \end{cases}$$

Symmetric Equations

$$t = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad \circledast$$

Ex. $P(5, 1, 3) = (x_0, y_0, z_0)$

Find all 3 equations of the line through P in the direction of

$$\vec{v} = \vec{i} + 4\vec{j} - 2\vec{k} = \langle 1, 4, -2 \rangle = \langle a, b, c \rangle$$

vector: $\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle = \langle 5 + t, 1 + 4t, 3 - 2t \rangle$

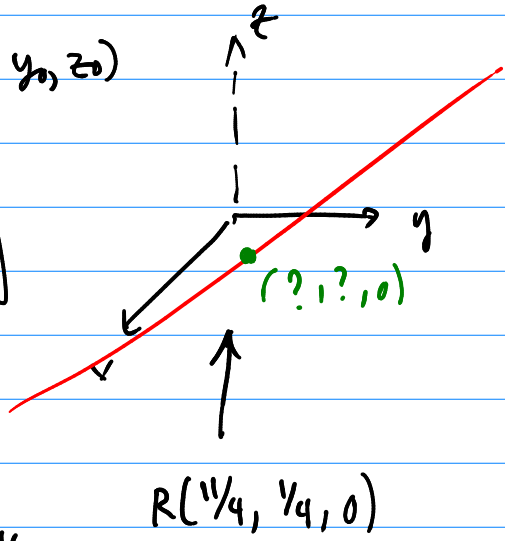
Param. $\begin{cases} x = 5+t \\ y = 1+4t \\ z = 3-2t \end{cases}$ Symm: $\frac{x-5}{1} = \frac{y-1}{4} = \frac{z-3}{-2}$

Ex. $A(2,4,-3)$ $B(3,-1,1)$ Any eqn of the line through both pts. Then, at what pt does l intersect the xy-plane?

$\vec{v} = \overrightarrow{AB} = \langle 1,-5,4 \rangle = \langle a,b,c \rangle$

$A(2,4,-3) = (x_0, y_0, z_0)$

$\vec{r}(t) = \langle 2+t, 4-5t, -3+4t \rangle$



$z=0$: $-3+4t=0$

$t = 3/4$

$x(3/4) = 2 + 3/4 = 11/4$ $y(3/4) = 4 - 15/4 = 1/4$

Ex. $P(x_1,y_1,z_1)$ $Q(x_2,y_2,z_2)$

Find an eqn of the line segment $\overline{PQ}$