M243 8/29

Test opens 9/4 @ 10:30 am
closes 9/6 @ 11:59 pm

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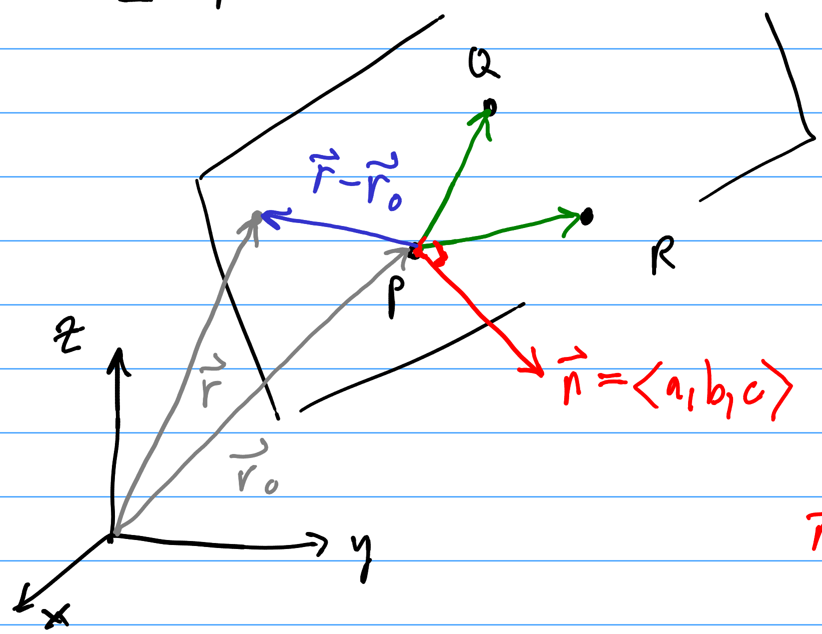
§12.5 Planes in space ($\mathbb{R}^3$)

We'll need 3 pieces of information: 3 non-collinear points.



$P(x_0, y_0, z_0)$

$\vec{r}_0 = \langle x_0, y_0, z_0\rangle$

$\vec{r} = \langle x, y, z\rangle$

$\vec{n} = \vec{PQ} \times \vec{PR}$

A normal vector $\vec{n}$ to a plane Π is a nonzero vector that is orthogonal to all vectors in the plane.

We obtain: the plane consists of all points for which

$$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0 \qquad (*)$$

Plug in: $\vec{n} = \langle a,b,c\rangle$ $\quad \vec{r} = \langle x,y,z\rangle$ $\quad \vec{r}_0 = \langle x_0, y_0, z_0\rangle$

$$\vec{r}-\vec{r}_0 = \langle x-x_0,\ y-y_0,\ z-z_0\rangle$$

$$\vec{n} = \langle a,b,c\rangle$$

$$\vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle a,b,c\rangle\cdot\langle x-x_0, y-y_0, z-z_0\rangle$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$ax+by+cz\underbrace{-(ax_0+by_0+cz_0)}_{+d}=0$$

$$\boxed{ax+by+cz+d=0} \qquad \boxed{d=-\vec{n}\cdot\vec{r}_0}$$

Ex. $P(1,3,2)$ $\quad Q(3,-1,6)$ $\quad R(5,2,0)$

Find an eq'n for Π. $\qquad \vec{r}_0=\langle 5,2,0\rangle$

$\vec{n}=\vec{PQ}\times\vec{PR}$

$\vec{PQ}=\langle 2,-4,4\rangle$

$\vec{PR}=\langle 4,-1,-2\rangle$

$$\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -4 & 4\\ 4 & -1 & -2\end{vmatrix}=\langle 12,20,14\rangle \sim \boxed{\langle 6,10,7\rangle=\vec{n}}$$

$$d=-\vec{n}\cdot\vec{r}_0=-\langle 6,10,7\rangle\cdot\langle 5,2,0\rangle$$

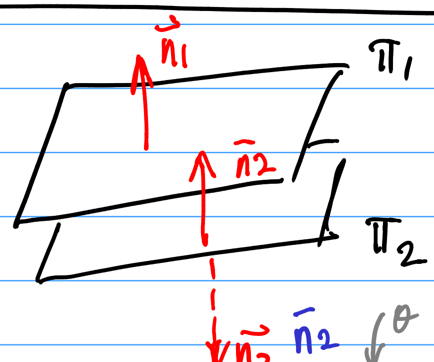
$$=-(30+20+0)=-50$$

Π: $ax+by+cz+d=0$

$6x+10y+7z-50=0$ ⊛

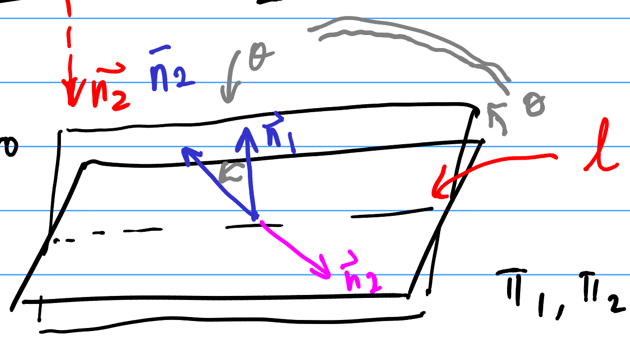
Π_1

Π_2



Parallel planes have parallel normal vectors.

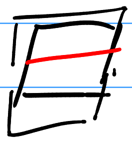
The angle between two intersecting planes is "the same" as the ∡ between normals



$0<\theta\le \pi/2$

Π_1: $x+y+z=1$

Π_2: $x-2y+3z=1$



Q1. Do they intersect? θ?

Not parallel, so they must int.

$\vec{n}_1 = \langle 1,1,1 \rangle$

$\vec{n}_2 = \langle 1,-2,3 \rangle$

$$\theta(\vec{n}_1, \vec{n}_2) = \arccos\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \, \|\vec{n}_2\|}\right)$$

$\|\vec{n}_1\| = \sqrt{1+1+1} = \sqrt{3}$

$\|\vec{n}_2\| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{n}_1 \cdot \vec{n}_2 = 1-2+3 = 2$

$$\theta = \arccos\left(\frac{2}{\sqrt{42}}\right) \approx 72^\circ$$

Q2. Find the eq'n of the line of intersection:

Point on both planes
Direction vector for l. $\}$

$$\vec{N} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = \langle 5, -2, -3 \rangle$$

To find the pt. we set $y, z = 0$, and get $x=1$

$$\vec{r}_0 = \langle 1, 0, 0 \rangle$$

$$\vec{r}(t) = \vec{r}_0 + t\vec{N} = \langle 1+5t, -2t, -3t \rangle$$