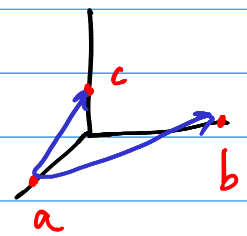


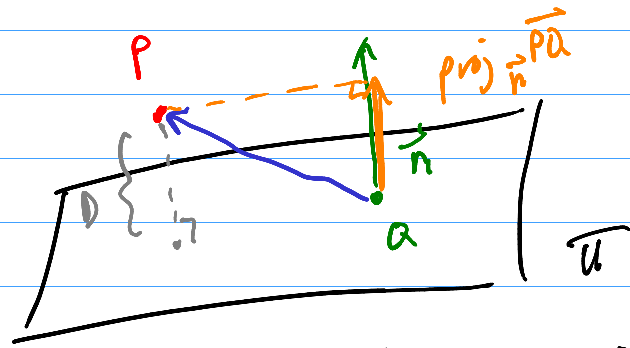
$\vec{PQ} = \langle -a, 0, c\rangle \qquad \vec{PR} = \langle -a, b, 0\rangle$

$\vec{n} = \vec{PQ} \times \vec{PR}$

$P(a,0,0)$

§12.5, planes

Given a plane Π and a point P not on the plane, what is the distance D from Π to P?



so $D = \|\mathrm{proj}_{\vec{n}} \vec{PQ}\| = \left\| \dfrac{\vec{n}\cdot\vec{PQ}}{\|\vec{n}\|} \dfrac{\vec{n}}{\|\vec{n}\|} \right\| = \dfrac{|\vec{n}\cdot\vec{PQ}|}{\|\vec{n}\|}$

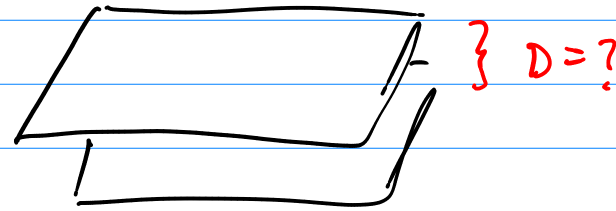
$\vec{n} = \langle a, b, c\rangle$

$\left.\begin{array}{l} Q = (x_0, y_0, z_0) \\ P = (x_1, y_1, z_1) \end{array}\right\} \quad \vec{QP} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$

$$\begin{aligned} \vec{QP}\cdot\vec{n} &= \langle a, b, c\rangle \cdot \langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle \\ &= a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) \\ &= ax_1 + by_1 + cz_1 - (ax_0 + by_0 + cz_0) \\ &= ax_1 + by_1 + cz_1 + d \end{aligned}$$

$\|\vec{n}\| = \sqrt{a^2 + b^2 + c^2}$ so, $\boxed{D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}}$ (*)

Ex. $\Pi_1: 10x + 2y - 2z = 5$ Parallel
$\Pi_2: 5x + y - z = 1$



Point: $(0,1,0)$ on Π_2.

Plane: Π_1: $a=10$, $b=2$, $c=-2$, $d=-5$

$$D = \frac{|10(0) + 2(1) + (-2)(0) - 5|}{\sqrt{100+4+4}} = \frac{|-3|}{\sqrt{108}} = \frac{3}{\sqrt{108}} = \frac{3}{\sqrt{9}\sqrt{12}} = \frac{1}{\sqrt{12}}$$

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§12.6 Quadric Surfaces

$ax + by + cz + d = 0$ Plane

General Equation: $Ax^2 + Bx + Cy^2 + Dy + Ez^2 + Fz + G = 0$

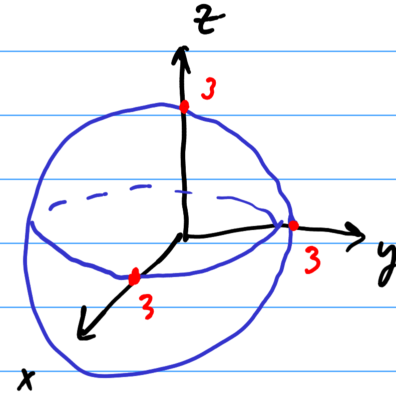
at least one of A, C, E are nonzero.

We can complete some squares to get formulas like:

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = d$$

Ex. $x^2+y^2+z^2=9$

sphere w/ radius 3



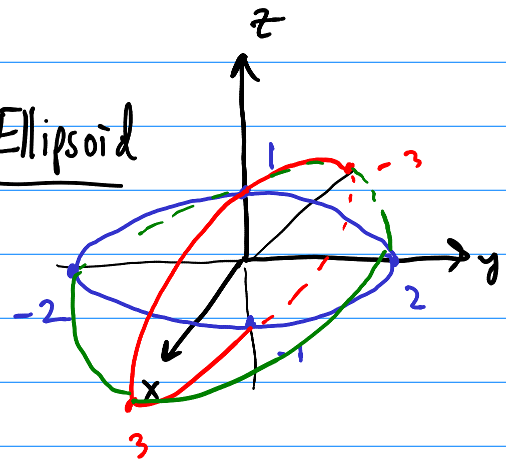
Ex. $\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2+z^2=1$

$x=0$: $\left(\frac{y}{2}\right)^2+z^2=1$

$y=0$: $\left(\frac{x}{3}\right)^2+z^2=1$

$z=0$: $\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2=1$

Ellipsoid



Ex. $z=x^2+2y^2$

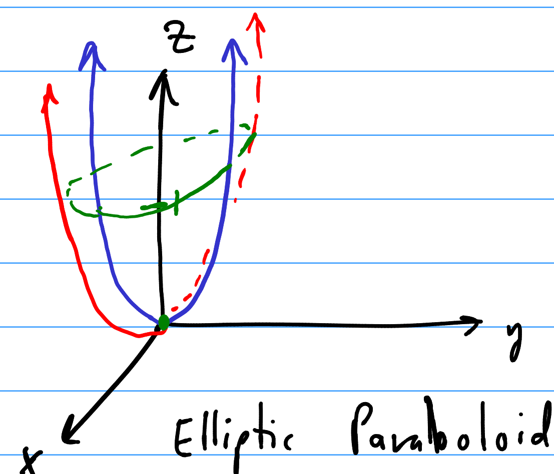
$x=0$: $z=2y^2$

$y=0$: $z=x^2$

$z=0$: $0=x^2+2y^2$

$z=-1$: $-1=x^2+2y^2$ X

$z=1$: $1=x^2+2y^2$



Elliptic Paraboloid

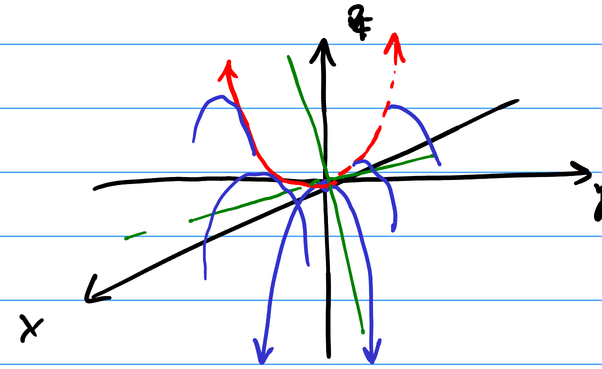
Ex. $z=x^2-2y^2$

$x=0$: $z=-2y^2$

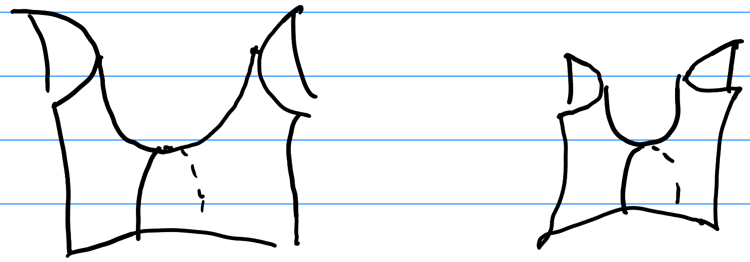
$y=0$: $z=x^2$

$z=0$: $0=x^2-2y^2$

$z=1$: $1=x^2-2y^2$



We get:



Hyperbolic Paraboloid or saddle surface