M243 9/25/18

$$\frac{1}{4}\int \frac{4\,dx}{x^2\sqrt{4x+1}} = \frac{1}{4}\int \frac{du}{\left(\frac{1}{4}u-\frac{1}{4}\right)^2\sqrt{u}} = 4\cdot 2\int \frac{du}{(u-1)^2\, 2\sqrt{u}}$$

$\boxed{u=4x+1} \rightsquigarrow 4x = u-1 \to x = \frac{1}{4}u - \frac{1}{4}$

$du = 4\,dx$

$\boxed{v=\sqrt{u}}$ $\quad u=v^2$

$dv = \frac{1}{2\sqrt{u}}\,du$

$$= 8\int \frac{dv}{(v^2-1)^2} \overset{\text{PFD}}{=} 8\int \underbrace{\frac{1}{(v+1)^2(v-1)^2}}\,dv$$

$v=\sqrt{4x+1}$

$$\frac{1}{(v+1)^2(v-1)^2} = \frac{A}{v+1} + \frac{B}{(v+1)^2} + \frac{C}{v-1} + \frac{D}{(v-1)^2}$$

$$= 8\int \frac{A}{v+1} + \frac{B}{(v+1)^2} + \frac{C}{v-1} + \frac{D}{(v-1)^2}\,dv$$

$$= 8\left(A\ln(v+1) - \frac{B}{v+1} + C\ln(v-1) - \frac{D}{v-1}\right) + K$$

$$= 8\left(A\ln\left(\sqrt{4x+1}+1\right) - \frac{B}{\sqrt{4x+1}+1} + C\ln\left(\sqrt{4x+1}-1\right) - \frac{D}{\sqrt{4x+1}-1}\right) + K$$

7.4.2: $\dfrac{x^4-2x^3+x^2+6x-5}{x^2-2x+1}$ PFD

$$\begin{array}{r} x^2 \\ x^2-2x+1 \overline{\smash{)}\, x^4-2x^3+x^2+6x-5} \\ -(x^4-2x^3+x^2) \\ \hline 0+6x-5 \end{array} \quad + \frac{6x-5}{x^2-2x+1}$$

$\implies x^2 + \dfrac{6x-5}{x^2-2x+1}$

x^2-1 . . .

PFD: $\dfrac{6x-5}{x^2-2x+1} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$

$$\frac{x^4-2x^3+x^2+6x-5}{x^2-2x+1} =$$

$6x-5 = A(x-1) + B$

$6x-5 = Ax + (B-A)$

$$\boxed{x^2 + \frac{6}{x-1} + \frac{1}{(x-1)^2}}$$

$A = 6$

$-5 = B - A = B - 6 \rightarrow B = 1$

$$\frac{x^4-2x^3+x^2+6x-5}{(x-1)^2}$$

$$\begin{array}{c|ccccc}
1 & 1 & -2 & 1 & 6 & -5 \\
 & \downarrow & 1 & -1 & 0 & 6 \\
\hline
1 & 1 & -1 & 0 & 6 & \boxed{1} \\
 & \downarrow & 1 & 0 & 0 & \downarrow \\
\hline
 & 1 & 0 & 0 & \boxed{6} &
\end{array}$$

$$x^2+0x+0+\frac{6+\frac{1}{x-1}}{x-1}$$

$$= x^2 + \frac{6}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{x^4-2x^3+x^2+6x-5}{(x-1)^2}\,dx$$

$$= \int x^2 + \frac{6}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$$= \frac{1}{3}x^3 + 6\ln|x-1| - \frac{1}{x-1} + C$$

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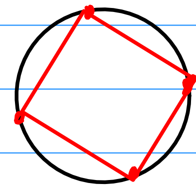
§8.1 - Arc Length

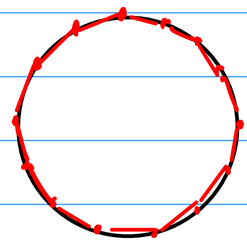
Motivating Example:



Find the circumference?

$C \approx P$

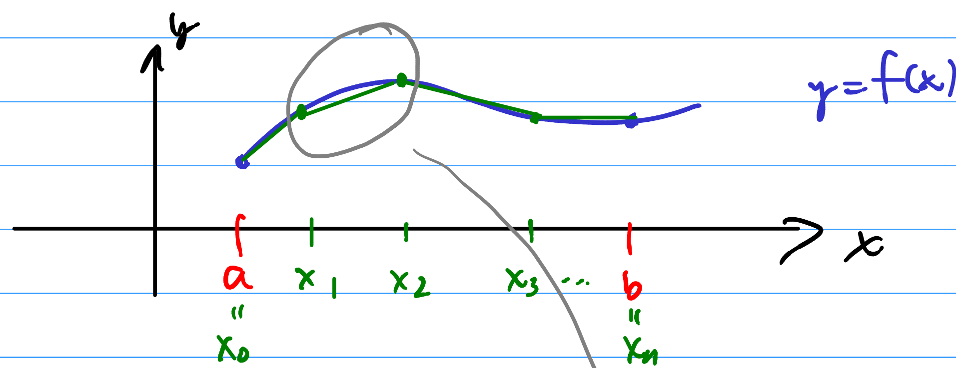
To get a better approx: choose more points.



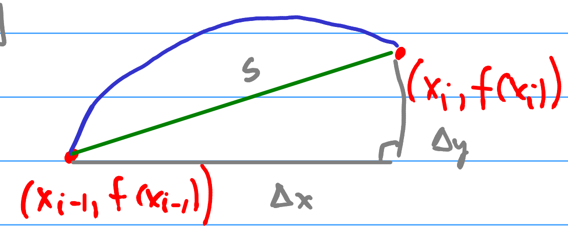
"closer"

In general, the circumference is $C = \lim_{n \to \infty} P_n$ where P_n is the perimeter of an n-gon.

For us, $y = f(x)$ be a smooth function on $[a,b]$.



$\ell \approx \underbrace{\sum \text{segment lengths}}_{\text{formula}}$



distance formula: $\ell \approx \sqrt{\Delta x^2 + \Delta y^2}$

$s = \sqrt{\Delta x^2 + \Delta y^2}$

The arc length is:

$$l \approx \sum_{n=0}^{N} \sqrt{\Delta x_n^2 + \Delta y_n^2} \qquad \text{so,} \quad l = \lim_{N\to\infty} \sum_{n=0}^{N} \sqrt{\Delta x_n^2 + \Delta y_n^2}$$

$$(*) \quad \boxed{l = \int_a^b \sqrt{dx^2 + dy^2}}$$

$$ds = \sqrt{dx^2+dy^2} = \sqrt{dx^2}\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \underbrace{\sqrt{1+(f'(x))^2}\,dx}$$

$$\frac{dy}{dx} = f'(x)$$

So $$l = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Ex. $y^2 = x^3$ $\quad (1,1) \to (4,8)$

solve for y: $y = \sqrt{x^3} = x^{3/2} = f(x)$ $\quad a=1,\ b=4$

$$l = \int_1^4 \sqrt{1+(f'(x))^2}\,dx$$

$f'(x) = (x^{3/2})' = \frac{3}{2}x^{1/2}$

$(f'(x))^2 = (\frac{3}{2}x^{1/2})^2 = \frac{9}{4}x$

$$l = \frac{4}{9}\int_1^4 \sqrt{1+\frac{9}{4}x}\ \frac{9}{4}\,dx$$

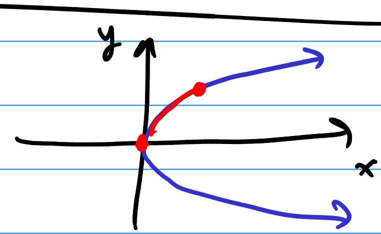
$u = 1+\frac{9}{4}x$ $\quad u(4) = 10$

$du = \frac{9}{4}dx$ $\quad u(1) = \frac{13}{4}$

$$= \frac{4}{9}\int_{13/4}^{10} \sqrt{u}\,du = \frac{4}{9}\int_{13/4}^{10} u^{1/2}\,du = \frac{4}{9}\frac{2}{3}u^{3/2}\Big|_{13/4}^{10} = \frac{8}{27}\left(10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

$$\approx 7.63$$

Ex. $\underbrace{x = y^2}$ from $(0,0)$ to $(1,1)$



$y = \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}}$

$$\boxed{l = \int_0^1 \sqrt{1+\frac{1}{4x}}\,dx}$$

OR

$x = f(y) = y^2$

$x' = 2y$

$l = \int_c^d \sqrt{1+(f'(y))^2}\, dy = \boxed{\frac{1}{2}\int_0^1 \sqrt{1+4y^2}\, 2\, dy}$

$1+(2y)^2$

put $2y = \tan\theta$

$2dy = \sec^2\theta\, d\theta$

$= \frac{1}{2}\underbrace{\int_{\square}^{\square} \sec^3\theta\, d\theta}$

$\underbrace{\sec\theta}_{u} \quad \underbrace{\sec^2\theta\, d\theta}_{dv}$