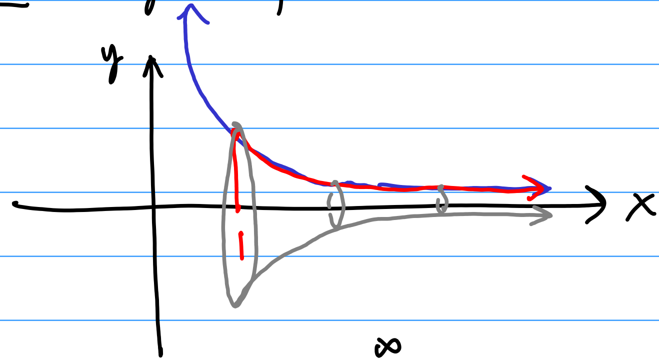
M243 9/28/18

Gabriel's Horn $y = 1/x$, $1 \le x < \infty$



Find the surface area: $\int_1^\infty 2\pi \frac{1}{x} \, ds$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$\frac{dy}{dx} = \frac{-1}{x^2}$ $\quad \left(\frac{dy}{dx}\right)^2 = \frac{1}{x^4}$ $\quad 1 + \left(\frac{dy}{dx}\right)^2 = \frac{x^4}{x^4} + \frac{1}{x^4} = \frac{x^4+1}{x^4}$

$ds = \frac{\sqrt{x^4+1}}{x^2} \, dx$

$A = \int_1^\infty 2\pi \frac{1x}{x^2} \frac{\sqrt{x^4+1}}{x^2} \, dx = \underbrace{\pi \int_1^\infty \frac{\boxed{\sqrt{u^2+1}}}{u^2} \, du}$

$\cancel{x^4+1 = u \quad du = 4x^3 \, dx}$

$u = x^2$
$du = 2x \, dx$

$u = \tan\theta$
$du = \underbrace{\sec^2\theta \, d\theta}$

$\theta = \arctan u$

$\theta = \arctan 1 = \pi/4$

$\theta(\infty) = \arctan(\infty) = \pi/2$

$\pi \int_{\pi/4}^{\pi/2} \frac{\sec^3\theta}{\tan^2\theta} \, d\theta = \pi \int_{\pi/4}^{\pi/2} \frac{\frac{1}{\cos^3\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} \, d\theta$

$= \pi \int_{\pi/4}^{\pi/2} \frac{1}{\cos\theta \sin^2\theta} \, d\theta$

$= \pi \int_{\pi/4}^{\pi/2} \underbrace{\sec\theta}_{u} \underbrace{\csc^2\theta \, d\theta}_{dv}$

$$\pi\int_{\pi/4}^{\pi/2} \sec\theta\,\csc^2\theta\,d\theta = \pi\left[-\sec\theta\cot\theta + \int \sec\theta\,\cancel{\tan\theta\cot\theta}\,d\theta\right]_{\pi/4}^{\pi/2}$$

$u = \sec\theta \qquad dv = \csc^2\theta\,d\theta$

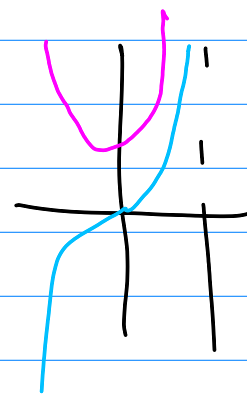
$du = \sec\theta\tan\theta\,d\theta \qquad v = -\cot\theta$

$$A = \pi\Big[-\sec\theta\cot\theta + \ln|\sec\theta + \tan\theta|\Big]\Big|_{\pi/4}^{\pi/2^-}$$

$$A = \pi\left(\underbrace{\lim_{\theta\to\pi/2^-}\left(-\sec\theta\cot\theta + \ln|\sec\theta+\tan\theta|\right)} + \sec\tfrac{\pi}{4}\cot\tfrac{\pi}{4} - \ln\left(\sec\tfrac{\pi}{4} + \tan\tfrac{\pi}{4}\right)\right)$$

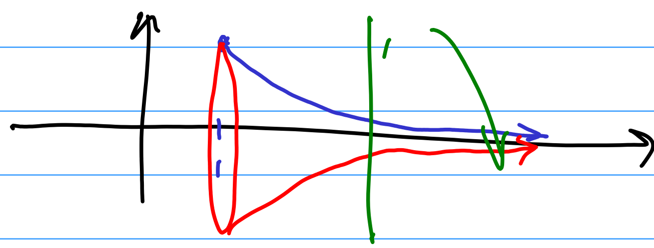
$$\lim_{\theta\to\pi/2^-} -\sec\theta\cot\theta = \lim_{\theta\to\pi/2^-} \frac{-1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta} = -1$$

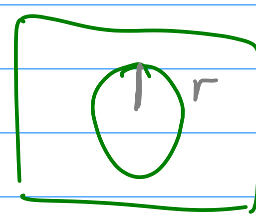
$$\lim_{\theta\to\pi/2^-} \ln|\sec\theta+\tan\theta| = \ln|+\infty+\infty| = +\infty$$



Surface Area is <u>divergent</u>, or infinite.

<u>Check the volume:</u>





$r = 1/x$

$$A = \pi r^2 = \pi(1/x)^2$$

$$V = \int_1^\infty A\,dx$$

$$= \int_1^\infty \pi\frac{1}{x^2}\,dx$$

$$= \pi\lim_{t\to\infty}\int_1^t \frac{1}{x^2}\,dx$$

$$= \pi\lim_{t\to\infty}\left(-\frac{1}{x}\Big|_1^t\right) = \pi\lim_{t\to\infty}\left(1-\frac{1}{t}\right) = \pi$$

8.2 Project "7.2 Project": Rotating on Slant.

1. Do it. (Turn in.)
2-4. Don't turn in.
5*. For $y = x + \sin x$ and line $y = x - 2$.
Write an integral that represents the surface area of the revolved surface for $0 \le x \le 2\pi$.
Then, use a computer to actually compute the value.

$\boxed{\text{Extra Credit: Friday, 5 October 2018}}$