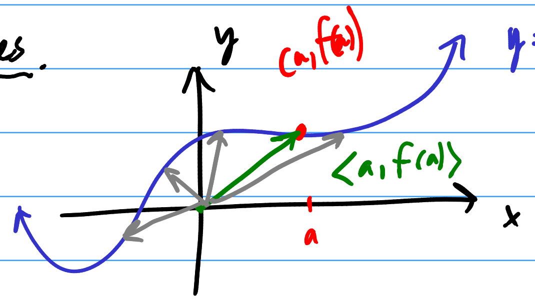
Chapter 9 – Parametrized Curves

Consider a vector $\langle x, y \rangle$ s.t. x and y are each functions of t.
Then the vector $\langle x(t), y(t) \rangle$ will trace out a curve in the xy-plane as t varies.

Examples.

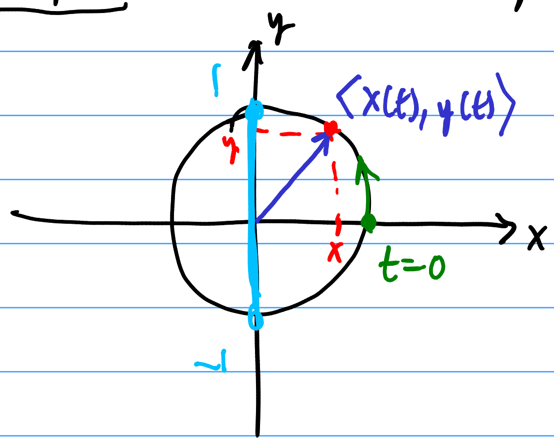


A (The) parametrization is:

$$\begin{cases} x(t) = t \\ y(t) = f(t) \end{cases}$$

or $\langle t, f(t) \rangle$

Example. The circle $x^2 + y^2 = 1$



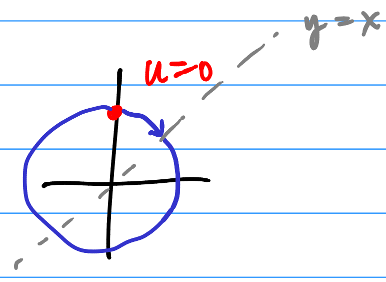
$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \end{cases}$$ standard parametrization

OR

$$\begin{cases} x(u) = \sin(u) \\ y(u) = \cos(u) \end{cases}$$

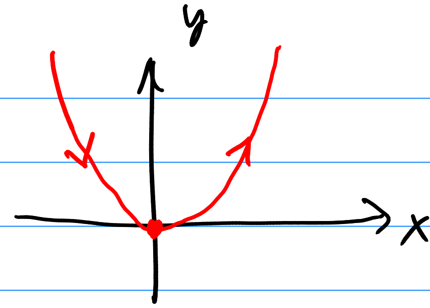
$\langle x(u), y(u) \rangle = \langle \sin u, \cos u \rangle$

$\langle x(0), y(0) \rangle = \langle 0, 1 \rangle$



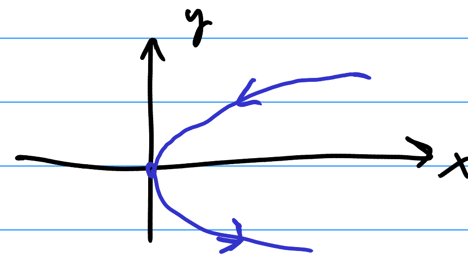
Ex. $\begin{cases} x = t \\ y = t^2 \end{cases}$

$y = x^2$



$\begin{cases} x = t^2 \\ y = t \end{cases}$

$x = y^2$

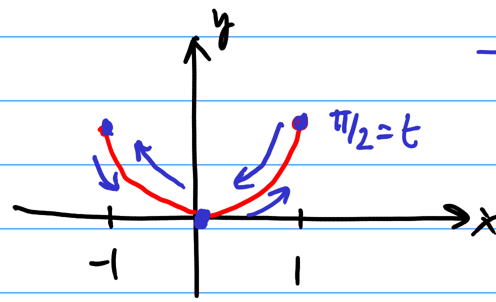


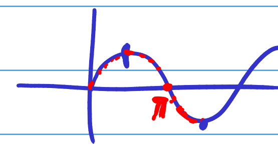
Ex. $\begin{cases} x = \sin t \\ y = \sin^2 t = (\sin t)^2 \end{cases}$

$y = x^2, \; -1 \le x \le 1$

because

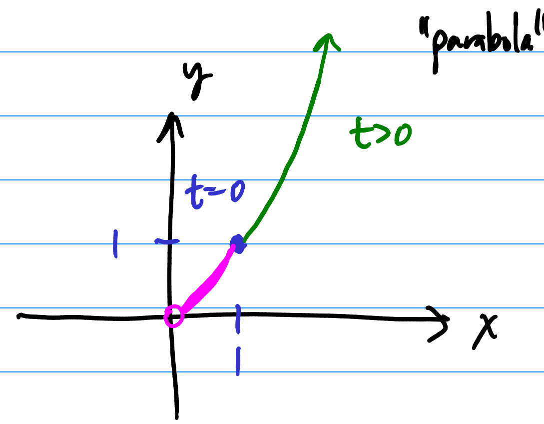
$[-1,1]$ is the range of $\sin t$.





Ex. $\begin{cases} x = e^t \\ y = e^{2t} = (e^t)^2 \end{cases}$

$y = x^2$

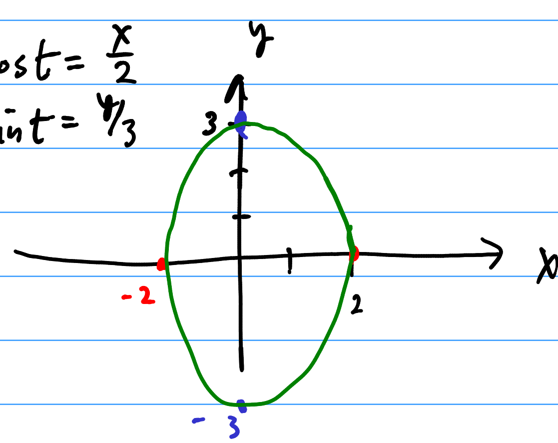


Ex. $\begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases} \rightarrow \begin{matrix} \cos t = \frac{x}{2} \\ \sin t = y/3 \end{matrix}$

$t = 0: \langle 2, 0 \rangle$

$t = \pi/2: \langle 0, 3 \rangle$

ellipse!



$$1 = \cos^2 t + \sin^2 t = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2$$

$$\boxed{\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1}$$

Not equal ⇒ ellipse !!

Ex. $x^2 - y^2 = 1$ hyperbola.

RHS
$$\begin{cases} x(t) = \cosh(t) \\ y(t) = \sinh(t) \end{cases}$$