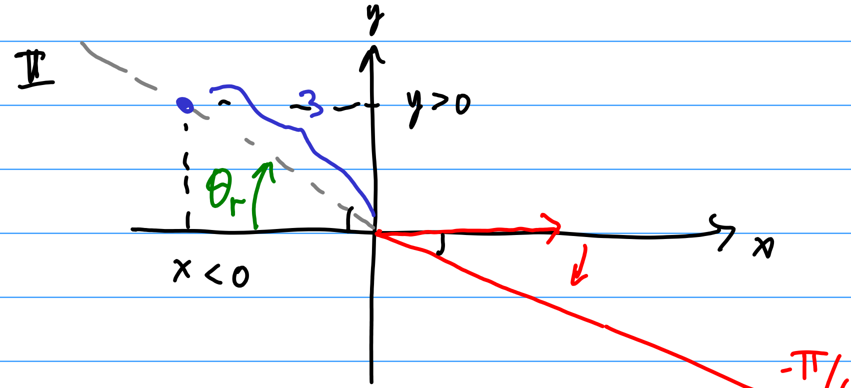
M243 25 Oct 18

10.3. #1 last box $\quad (-3, -\pi/6)$ Find rect. coord's.



$\theta_r = \pi/6$

$$\begin{cases} x = -3\cos(\pi/6) = -3\frac{\sqrt{3}}{2} \\ y = 3\sin(\pi/6) = 3\,{}^{1}/_{2} \end{cases}$$

10.3.17

$r(\theta) = \underbrace{5 + 2\cos\theta}$ $\quad \theta = \pi/3 \quad$ Find dy/dx

$$\begin{cases} x(\theta) = r(\theta)\cos(\theta) = 5\cos\theta + 2\cos^2\theta \\ y(\theta) = r(\theta)\sin(\theta) = 5\sin\theta + 2\sin\theta\cos\theta \end{cases}$$

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$$

$\dot{y} = 5\cos\theta - 2\sin^2\theta + 2\cos^2\theta$

$\dot{y}(\pi/3) = \frac{+5\cdot 1}{2} - 2\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) = \frac{5}{2} - \frac{3}{2} + \frac{1}{2} = 3/2$

$\dot{x} = -5\sin\theta - 4\cos\theta\sin\theta$

$\dot{x}(\pi/3) = -5\frac{\sqrt{3}}{2} - 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{5\sqrt{3}}{2} - \frac{2\sqrt{3}}{2} = -\frac{7\sqrt{3}}{2}$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \frac{3/\cancel{2}}{-7\sqrt{3}/\cancel{2}} = \boxed{-\frac{\sqrt{3}}{7}}$$

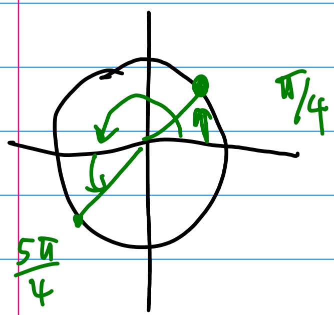
10.3.18 $\quad$ Find pts on $r(\theta) = e^\theta$ where $\frac{dy}{dx} = \begin{cases} 0 & \text{hor.} \\ \pm\infty & \text{vertical} \end{cases}$

H: $\quad \dot{y} = 0$

V: $\quad \dot{x} = 0$

$$\begin{cases} x(\theta) = r(\theta)\cos\theta = \underbrace{e^\theta\cos\theta} \\ y(\theta) = e^\theta\sin\theta \end{cases}$$

V $\quad \dot{x} = e^{\theta}\cos\theta - e^{\theta}\sin\theta = 0 \quad \rightarrow \quad \underline{e^{\theta}}\,(\underline{\cos\theta - \sin\theta}) = 0$



$\cos\theta - \sin\theta = 0 \qquad 0 \leq \theta \leq 2\pi$

$\cos\theta = \sin\theta$

$1 = \tan\theta$

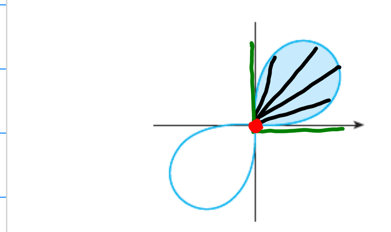
$\arctan 1 = \theta \qquad \underbrace{\theta = \pi/4, \frac{5\pi}{4}}$

$r(\pi/4) = e^{\pi/4} \qquad (e^{\pi/4}, \pi/4)$

$r(5\pi/4) = e^{5\pi/4} \qquad (e^{5\pi/4}, 5\pi/4)$

} vertical

10.4.3.


Find the area of the shaded region.

$r^2 = \sin(2\theta)$

$dA = \frac{1}{2} r^2 \, d\theta$

$A = \int_{\square}^{\square} dA = \frac{1}{2} \int_{\boxed{0}}^{\boxed{\pi/2}} \sin(2\theta)\, d\theta$

$r^2(0) = \sin(2\cdot 0) = \sin 0 = 0$

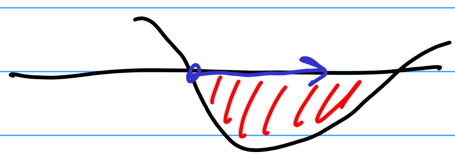
$r^2(\pi/2) = \sin(2\cdot \pi/2) = \sin\pi = 0$

$= \frac{1}{2} \cdot -\frac{1}{2} \cos(2\theta) \Big|_0^{\pi/2}$

$\sin(2\theta) = 0$

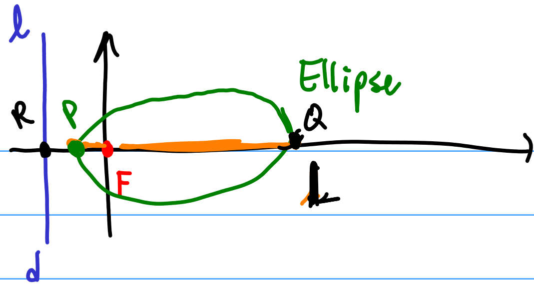
$= -\frac{1}{4}\left(\cos(\pi) - \cos(0)\right)$

$= -\frac{1}{4}(-1-1) = \boxed{\frac{1}{2}}$



10.5/6 #10

$\boxed{e} = 0.09\ldots \qquad$ (semi-)major axis length $= L$



$$\boxed{\frac{d(F,P)}{d(l,P)} = e}$$

$$d(F,P) = e\, d(l,P)$$

$$r = \frac{e \cdot d}{1 - e\cos\theta}$$

Job: Find $d = \overline{RF}$

$$\overline{PQ} = \overline{PF} + \overline{FQ} \Rightarrow \overline{FQ} = \overline{PQ} - \overline{PF}$$

$$L = d(P,Q)$$

$$d = d(P,l) + d(P,F) = (1+e)\, d(P,l)$$

$$d(l,Q) = e\, d(F,Q) = e\big(\underbrace{d(P,Q)}_{L} - \underbrace{d(P,F)}\big)$$

$$\overline{RF} = \overline{RP} + \overline{PF} = d(P,l) + d(P,F) = (1+e)\, d(P,l) = (1+e)\,\overline{RP}$$

$$\frac{1}{e}\overline{FQ} = \overline{RQ} = \overline{RP} + \overline{PQ} = \overline{RP} + L$$

$$\overline{RP} = \frac{1}{e}\overline{FQ} - L$$

$$\overline{RP} = \frac{1}{1+e}\overline{RF} = \frac{d}{1+e} \qquad \text{so} \qquad d = (1+e)\,\overline{RP}$$

... tell me tomorrow.