M243 | 31 Oct '18

§9.2 – Vector Fields and Euler's Method

Recall: $\frac{dy}{dx} = f(x,y)$ e.g., $\frac{dy}{dx} = 3 - 2x - \frac{1}{2}y$

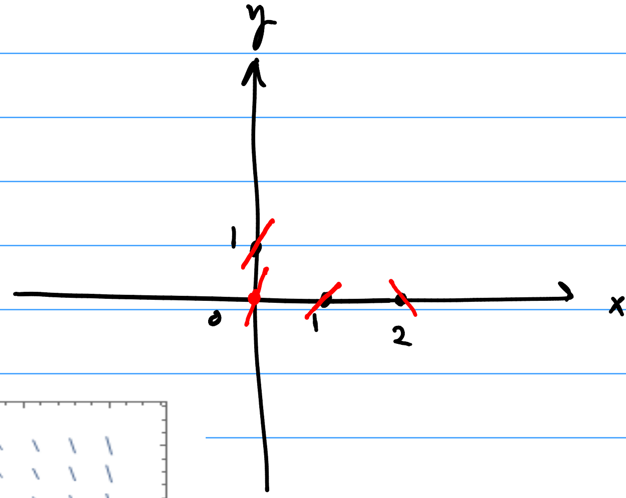
Goal: Find the unknown function $y = y(x)$ and/or find the solution curves in the xy-plane.

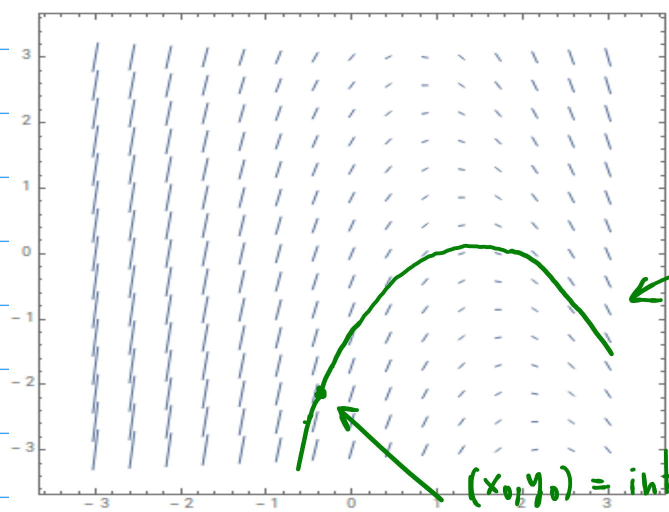
Def'n. For every DE $\frac{dy}{dx} = f(x,y)$, there is a corresponding slope field (or vector field).

At each point (x_0, y_0), $\frac{dy}{dx} = f(x_0, y_0)$ is determined by the DE.

Ex. $y' = 3 - 2x - \frac{1}{2}y$

(x,y)	y'
$(0,0)$	3
$(1,0)$	$3-2(1) = 1$
$(2,0)$	$3-2(2) = -1$
$(0,1)$	$3-\frac{1}{2} = \frac{5}{2}$



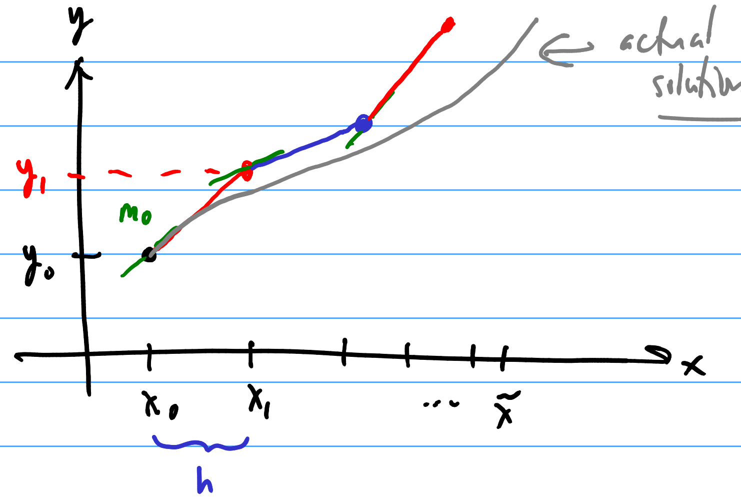


Sample solution curve

(x_0, y_0) = initial data

This leads to $\underline{\text{Euler's Method}}$

IVP $\begin{cases} dy/dx = f(x,y) \leftarrow \\ (x_0, y_0) \end{cases}$



Goal: Approximate the curve on $[x_0, \tilde{x}]$

$x_1 = x_0 + h$

$y_1 = ?$

$\underbrace{m_0 = f(x_0, y_0)}$

Eq'n of line: $y - y_0 = m_0(x - x_0)$

$y = y_0 + f(x_0, y_0)(x - x_0)$

So $y_1 = y_0 + f(x_0, y_0)\underbrace{(x_1 - x_0)}_{h}$

$$\boxed{\text{Euler's Method Algorithm} \quad \boxed{\begin{aligned} y_{k+1} &= y_k + h \cdot m_k \\ m_k &= f(x_k, y_k) \end{aligned}}}$$