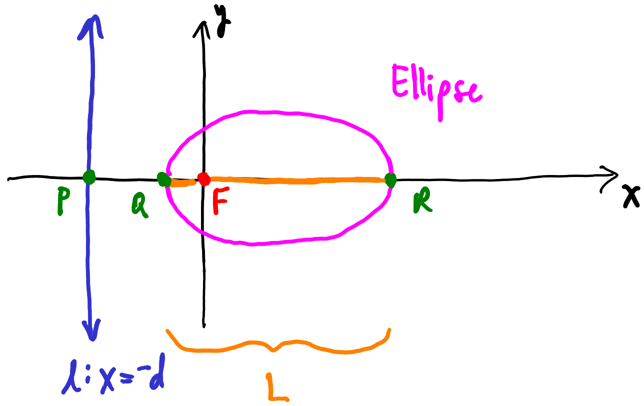
M243 25 Oct 18

The Ellipse Problem, Revisited

Let eccentricity be $0<e<1$, and major radius be L. The problem is to determine the equation of the directrix.

$r(\theta)=\dfrac{e\cdot d}{1-e\cos\theta}$

We must find d in terms of L and e.



$\overline{QR}=L$ (given)

$\overline{PF}=d$ (to be determined)

$\overline{PF}=\overline{PQ}+\overline{QF}=\overline{PQ}+e\,\overline{PQ}=(1+e)\overline{PQ}$

$$\begin{array}{rl} & e\,\overline{PQ} = \overline{QF} \\ + & e\,\overline{PR} = \overline{FR} \\ \hline \end{array} \qquad \begin{cases} \overline{QR}=\overline{QF}+\overline{FR} \\ \overline{PR}=\overline{PQ}+L \end{cases}$$

$$e(\overline{PQ}+\overline{PR})=\overline{QF}+\overline{FR}=L$$

$$e(2\overline{PQ}+L)=L \Rightarrow \overline{PQ}=\frac{1}{2}\left(\frac{L}{e}-L\right)$$

So,

$$d=\frac{1}{2}\left(\frac{L-eL}{e}\right)(1+e)$$

$$=\frac{L}{2}\left(\frac{1-e^2}{e}\right)$$

$$(*)\ \boxed{d=\frac{L}{2}\left(\frac{1}{e}-e\right)}$$

So the formula for the ellipse is:

$$r(\theta)=\frac{e\cdot d}{1-e\cdot\cos\theta}=\frac{\cancel{e}\cdot\frac{L}{2}\left(\frac{1-e^2}{\cancel{e}}\right)}{1-e\cdot\cos\theta}=\boxed{\frac{L(1-e^2)}{2-2e\cdot\cos\theta}=r(\theta)}$$

Plug in L and e when given to get the equation.