Calc II: Unit IV Review Guide

1. a) $x+y=2$

$r\cos\theta + r\sin\theta = 2$

$r(\cos\theta + \sin\theta) = 2$

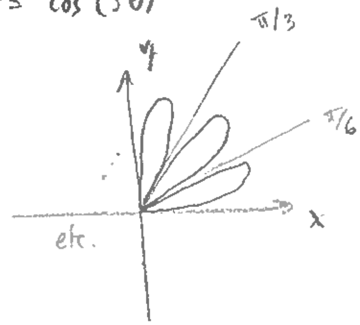
$r = \dfrac{2}{\cos\theta + \sin\theta}$

b.) $x^2+y^2=2$

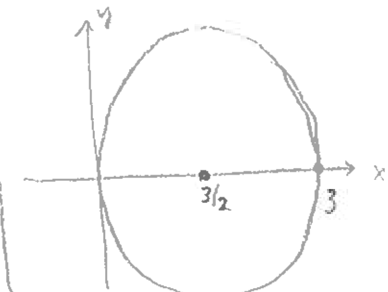
$r^2 = 2$

$r = \sqrt{2}$.

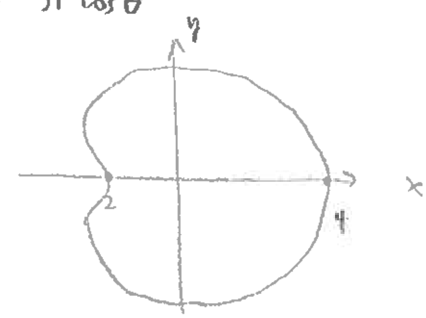
2. a) $r = \cos(3\theta)$



b) $r = 3\cos\theta$



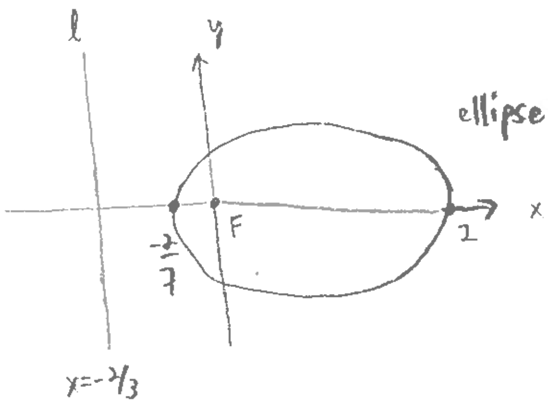
c) $r = 3 + \cos\theta$



3. $r = \dfrac{2}{4-3\cos\theta} = \dfrac{1/2}{1-3/4\cos\theta} = \dfrac{3/4 \cdot 2/3}{1-3/4\cos\theta}$

directrix: $x = d = -2/3$

eccentricity: $e = 3/4$



$$\text{Area} = \frac{1}{2}\int_0^{2\pi} \left(\frac{2}{4-3\cos\theta}\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{4}{16-24\cos\theta+9\cos^2\theta}\, d\theta \;. \;.$$

Don't worry about the area and arc length via integration of this formula for the in-class exam.

You could use geometry to compute these, or reparametrize in a more standard way.

4. $\begin{cases} x = \ln t \\ y = 1+t^2 \end{cases}$, $\begin{cases} \dot{x} = 1/t \\ \dot{y} = 2t \end{cases}$ $\quad \frac{dy}{dx} = \frac{2t}{1/t} = 2t^2$, at $t=1$, $\frac{dy}{dx} = 2$.

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5. $\begin{cases} x = t + \sin t \\ y = t - \cos t \end{cases}$, $\begin{cases} \dot{x} = 1 + \cos t \\ \dot{y} = 1 + \sin t \end{cases}$

$\frac{dy}{dx} = \frac{1+\sin t}{1+\cos t}$, $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(1+\cos t)\cos t + (1+\sin t)\sin t}{(1+\cos t)^2} = \frac{\cos t + \cos^2 t + \sin t + \sin^2 t}{1 + 2\cos t + \cos^2 t}$

so, $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{1 + \cos t + \sin t}{(1+\cos t)^3}$

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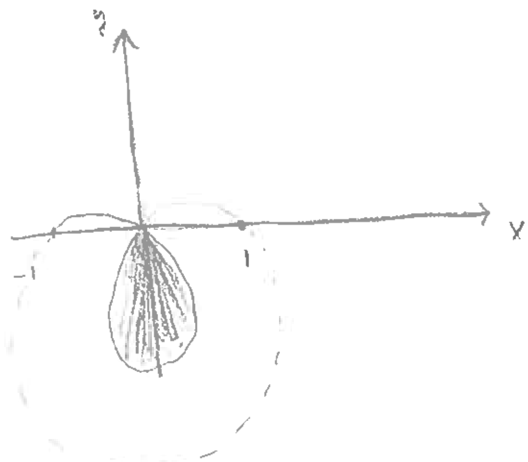
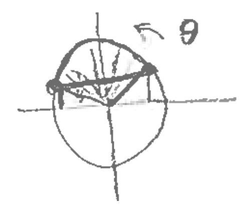
6. $r = 1 - 3\sin\theta$



$1 - 3\sin\theta = 0$

$\sin\theta = \frac{1}{3}$

$\alpha = \arcsin(1/3)$



$$A = \frac{1}{2}\int_{\alpha}^{\pi-\alpha} (1-3\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{\alpha}^{\pi-\alpha} 1 - 6\sin\theta + 9\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_{\alpha}^{\pi-\alpha} 1 \, d\theta - 3\int_{\alpha}^{\pi-\alpha} \sin\theta \, d\theta + \frac{9}{4}\int_{\alpha}^{\pi-\alpha} 1 + \cos(2\theta) \, d\theta$$

$$= \frac{1}{2}(\pi - \alpha - \alpha) + 3\left(\cos(\pi-\alpha) - \cos(\alpha)\right) + \frac{9}{4}(\pi - \alpha - \alpha) + \frac{9}{8}\left(\sin(2\pi - 2\alpha) - \sin(2\alpha)\right)$$

$$= \cancel{\frac{1}{2}\pi} - \cancel{\alpha} - 6\cos(\alpha) + \cancel{\frac{9}{4}\pi} - \cancel{\frac{9}{2}\alpha} + \frac{9}{8}\cdot 2\sin(\pi-\alpha)\cos(\pi-\alpha) - \frac{9}{8}\cdot 2\sin(\alpha)\cos(\alpha)$$

$$= \frac{11}{4}\pi - \frac{11}{2}\alpha - 6\cos(\alpha) + \frac{9}{2}\sin(\alpha)\cos(\alpha)$$

$\sin\alpha = \frac{1}{3}$, $\cos\alpha = \frac{\sqrt{8}}{3}$ $\quad = \frac{11\pi}{4} - \frac{11}{2}\arcsin\left(\frac{1}{3}\right) - 4\sqrt{2} + \frac{9}{2}\cdot\frac{2\sqrt{2}}{9}$

$$\boxed{A = \frac{11}{4}\pi - \frac{11}{2}\arcsin\left(\frac{1}{3}\right) - 3\sqrt{2}}$$

7. $\begin{cases} x = 2+3t \\ y = \cosh(3t) \end{cases}$

$$s = \int_0^1 \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

$$= \int_0^1 \sqrt{9 + 9\sinh^2 3t}\, dt$$

$$= \int_0^1 3\cosh 3t\, dt$$

$$= 3\sinh(3) - 3\sinh(0)$$

$$= 3\left(\frac{e^3 - 1/e^3}{2}\right) - 3\left(\cancel{\frac{1-1}{2}}\right)$$

$$= \frac{3(e^6-1)}{2e^3} \approx 27.2314$$

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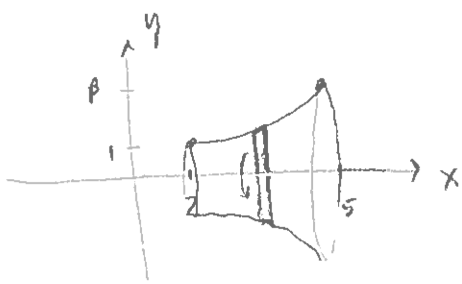
8. $\begin{cases} x = 2+3t \\ y = \cosh(3t) \end{cases}$



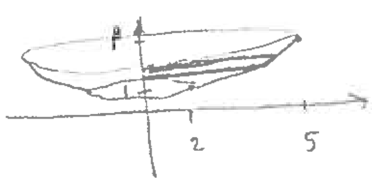
$\cosh(0) = 1$ $\quad \cosh(3) = \frac{e^6+1}{2e^3} = \beta$

$$A = 2\pi \int_0^1 R\, ds = 2\pi \int_0^1 y\sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

$$= 2\pi \int_0^1 \cosh(3t) \cdot 3\cosh(3t)\, dt$$

$$= 6\pi \int_0^1 \cosh^2(3t)\, dt = \frac{(e^{12} + 12e^6 - 1)\pi}{4e^6} \approx 326.275$$

9.



$$A = 2\pi \int_0^1 x\sqrt{\dot{x}^2+\dot{y}^2}\, dt = 2\pi \int_0^1 3 \cdot 3\cosh(3t)\, dt$$

$$= 6\pi \sinh(3) \approx 188.8325$$

10. directrix: $r = 4\sec\theta \Rightarrow r = \frac{4}{\cos\theta} \Rightarrow r\cos\theta = 4 \Rightarrow x = 4$

so $d = +4$, $e = 1$

$$\boxed{r(\theta) = \frac{4}{1+\cos\theta}}$$

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11. directrix is $x = +4$ again. $e = 1/3$.

$$r(\theta) = \frac{4 \cdot \frac{1}{3}}{1 + \frac{1}{3}\cos\theta} = \frac{4}{3 + \cos\theta}$$

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12. $x = +4 = d$, $e = 3$

$$r(\theta) = \frac{4 \cdot 3}{1 + 3\cos\theta} = \frac{12}{1 + 3\cos\theta}$$

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13. Check Slack.