Name:_____
M243: Calculus II (Spring 2018)
Instructor: Justin Ryan
Chapter 12 Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]
*Neatly write **T** if the statement is always true, and **F** otherwise.*

_____**1.** If $\mathbf{x} = \langle x_1, x_2 \rangle$ and $\mathbf{y} = \langle y_1, y_2 \rangle$, then $\mathbf{x} \cdot \mathbf{y} = \langle x_1 y_1, x_2 y_2 \rangle$.

_____**2.** The vector $\langle 3, -1, 2 \rangle$ is parallel to the plane $6x - 2y + 4z = 1$.

_____**3.** If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$, then $|\mathbf{x} \cdot \mathbf{y}| \leq \|\mathbf{x}\| \, \|\mathbf{y}\|$.

_____**4.** If $\mathbf{x} \times \mathbf{y} = \mathbf{0}$, then either $\mathbf{x} = \mathbf{0}$ or $\mathbf{y} = \mathbf{0}$.

_____**5.** The volume of the parallelepiped determined by vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ in $\mathbb{R}^3$ is given by $\text{vol} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

Part II: Multiple Choice [5 points each]
Select the best answer and write its corresponding letter neatly on the given line.

_____**6.** Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$. Then $(\mathbf{x} \cdot \mathbf{y}) \times \mathbf{z}$ is

A. a number **B.** a vector

C. a function **D.** undefined

_____**7.** Write the equation of the sphere in standard form, $x^2 + y^2 + z^2 - 2x + 2y - 4z = -2$.

A. $(x+1)^2 + (y-1)^2 + (z+2)^2 = 4$ **B.** $(x-1)^2 + (y+1)^2 + (z-2)^2 = 4$

C. $(x-1)^2 + (y+1)^2 + (z-2)^2 = -2$ **D.** $x^2 + y^2 + z^2 = 4$

8–11. Consider the vectors in $\mathbb{R}^3$.

$$\mathbf{x} = \langle 1, -2, 3 \rangle, \quad \mathbf{y} = \langle 0, -6, -8 \rangle, \quad \mathbf{z} = \langle -1, 1, 10 \rangle$$

_____**8.** Compute $\|\mathbf{y}\|$.

A. 10 **B.** $\sqrt{10}$

C. $\sqrt{14}$ **D.** 14

_____**9.** Compute $\mathbf{x} \cdot \mathbf{z}$.

A. 33 **B.** $\langle -1, -2, 30 \rangle$

C. 27 **D.** $\sqrt{27}$

_____**10.** Compute $\mathbf{y} \times \mathbf{z}$.

A. $\langle -52, 8, -6 \rangle$ **B.** $-52\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$

C. $52\mathbf{i} - 8\mathbf{j} + 6\mathbf{k}$ **D.** $\langle 52, 8, 6 \rangle$

_____**11.** Compute $\text{proj}_{\mathbf{y}}\mathbf{x}$.

A. $\langle 0, \frac{18}{25}, \frac{24}{25} \rangle$ **B.** $\langle -\frac{3}{25}, \frac{6}{25}, -\frac{9}{25} \rangle$

C. $\langle -\frac{6}{7}, \frac{12}{7}, -\frac{18}{7} \rangle$ **D.** $\langle 0, \frac{36}{7}, \frac{48}{7} \rangle$

_____**12.** Find a unit vector in the same direction as $\mathbf{x} = -4\mathbf{i} + 3\mathbf{k}$.

A. $\langle -\frac{4}{5}, \frac{3}{5}, 0 \rangle$ **B.** $\langle -\frac{4}{5}, 0, \frac{3}{5} \rangle$

C. $\langle -4, 3 \rangle$ **D.** $\langle -20, 0, 15 \rangle$

_____**13.** If $\|\mathbf{x}\| = 3$, $\|\mathbf{y}\| = 4$, and $\theta = \pi/3$, compute $\mathbf{x} \cdot \mathbf{y}$.

A. $6\sqrt{2}$ **B.** $\langle 3, 4, \frac{\pi}{3} \rangle$

C. 6 **D.** $6\sqrt{3}$

_____**14.** Find the area of the triangle with vertices $P(1,1,0)$, $Q(-1,0,1)$, and $R(1,1,1)$.

A. $\sqrt{5}$ **B.** 3

C. $\frac{1}{2}$ **D.** $\frac{\sqrt{5}}{2}$

_____**15.** Compute the angle between the planes

$$\Pi_1 : \quad x + z = 12$$
$$\Pi_2 : \quad y - z = 32$$

A. $\frac{\pi}{3}$ **B.** $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$ **D.** $\frac{3\pi}{4}$

Part III: Written Problems [10 points each]
Complete all problems, showing enough work.

16. Prove the *Parallelogram Law*: For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$,

$$\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2\|\mathbf{x}\|^2 + 2\|\mathbf{y}\|^2.$$

17. Suppose A, B, C are vertices of a triangle. Find $\overline{AB} + \overline{BC} + \overline{CA}$. Show enough work. If you include a picture, be sure to also include an explanation of what the picture means/says.

18. Find an equation of the plane passing through the points $P(1,2,3)$, $Q(4,0,-1)$, and $R(2,-4,-2)$.

19. Find the distance between the given point and the given plane.

$$\begin{cases} P(-1,3,-2) \\ \Pi:\ 3x+2y+6z=5 \end{cases}$$

20. Find all three equations (vector, parametric, symmetric) of the line passing through $P(0,\frac{1}{2},1)$ and $Q(3,1,-4)$.