Name: Key

M243: Calculus II (Spring 2018)
Instructor: Justin Ryan
Chapter 12 Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]
*Neatly write **T** if the statement is always true, and **F** otherwise.*

F **1.** If $\mathbf{x} = \langle x_1, x_2 \rangle$ and $\mathbf{y} = \langle y_1, y_2 \rangle$, then $\mathbf{x} \cdot \mathbf{y} = \langle x_1 y_1, x_2 y_2 \rangle$.

F **2.** The vector $\langle 3, -1, 2 \rangle$ is parallel to the plane $6x - 2y + 4z = 1$.

T **3.** If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$, then $|\mathbf{x} \cdot \mathbf{y}| \leq \|\mathbf{x}\| \, \|\mathbf{y}\|$.

F **4.** If $\mathbf{x} \times \mathbf{y} = \mathbf{0}$, then either $\mathbf{x} = \mathbf{0}$ or $\mathbf{y} = \mathbf{0}$.

T **5.** The volume of the parallelepiped determined by vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ in $\mathbb{R}^3$ is given by $\text{vol} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

Part II: Multiple Choice [5 points each]
Select the best answer and write its corresponding letter neatly on the given line.

D **6.** Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$. Then $(\mathbf{x} \cdot \mathbf{y}) \times \mathbf{z}$ is

A. a number **B.** a vector

C. a function **D.** undefined

B **7.** Write the equation of the sphere in standard form, $x^2 + y^2 + z^2 - 2x + 2y - 4z = -2$.

A. $(x+1)^2 + (y-1)^2 + (z+2)^2 = 4$ **B.** $(x-1)^2 + (y+1)^2 + (z-2)^2 = 4$

C. $(x-1)^2 + (y+1)^2 + (z-2)^2 = -2$ **D.** $x^2 + y^2 + z^2 = 4$

8–11. Consider the vectors in $\mathbb{R}^3$.

$$\mathbf{x} = \langle 1, -2, 3\rangle, \quad \mathbf{y} = \langle 0, -6, -8\rangle, \quad \mathbf{z} = \langle -1, 1, 10\rangle$$

__A__ **8.** Compute $\|\mathbf{y}\|$.

A. 10 **B.** $\sqrt{10}$

C. $\sqrt{14}$ **D.** 14

__C__ **9.** Compute $\mathbf{x} \cdot \mathbf{z}$.

A. 33 **B.** $\langle -1, -2, 30\rangle$

C. 27 **D.** $\sqrt{27}$

__A__ **10.** Compute $\mathbf{y} \times \mathbf{z}$.

A. $\langle -52, 8, -6\rangle$ **B.** $-52\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$

C. $52\mathbf{i} - 8\mathbf{j} + 6\mathbf{k}$ **D.** $\langle 52, 8, 6\rangle$

__A__ **11.** Compute $\text{proj}_{\mathbf{y}}\mathbf{x}$.

A. $\langle 0, \frac{18}{25}, \frac{24}{25}\rangle$ **B.** $\langle -\frac{3}{25}, \frac{6}{25}, -\frac{9}{25}\rangle$

C. $\langle -\frac{6}{7}, \frac{12}{7}, -\frac{18}{7}\rangle$ **D.** $\langle 0, \frac{36}{7}, \frac{48}{7}\rangle$

B 12. Find a unit vector in the same direction as $\mathbf{x} = -4\mathbf{i} + 3\mathbf{k}$.

A. $\langle -\frac{4}{5}, \frac{3}{5}, 0\rangle$ **B.** $\langle -\frac{4}{5}, 0, \frac{3}{5}\rangle$

C. $\langle -4, 3\rangle$ **D.** $\langle -20, 0, 15\rangle$

C 13. If $\|\mathbf{x}\| = 3$, $\|\mathbf{y}\| = 4$, and $\theta = \pi/3$, compute $\mathbf{x}\cdot\mathbf{y}$.

A. $6\sqrt{2}$ **B.** $\langle 3, 4, \frac{\pi}{3}\rangle$

C. 6 **D.** $6\sqrt{3}$

D 14. Find the area of the triangle with vertices $P(1,1,0)$, $Q(-1,0,1)$, and $R(1,1,1)$.

A. $\sqrt{5}$ **B.** 3

C. $\frac{1}{2}$ **D.** $\frac{\sqrt{5}}{2}$

B 15. Compute the angle between the planes

$$\Pi_1: \quad x + z = 12$$
$$\Pi_2: \quad y - z = 32$$

A. $\frac{\pi}{3}$ **B.** $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$ **D.** $\frac{3\pi}{4}$

Part III: Written Problems [10 points each]
Complete all problems, showing enough work.

16. Prove the *Parallelogram Law*: For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$,

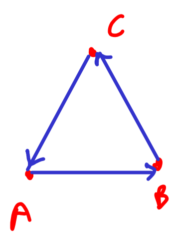
$$\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2\|\mathbf{x}\|^2 + 2\|\mathbf{y}\|^2.$$

$$\|\vec{x}+\vec{y}\|^2 = (\vec{x}+\vec{y})\cdot(\vec{x}+\vec{y}) = \vec{x}\cdot\vec{x} + \vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{x} + \vec{y}\cdot\vec{y}$$
$$= \|\vec{x}\|^2 + 2\,\vec{x}\cdot\vec{y} + \|\vec{y}\|^2$$

similarly, $\|\vec{x}-\vec{y}\|^2 = \|\vec{x}\|^2 - 2\vec{x}\cdot\vec{y} + \|\vec{y}\|^2$

Therefore,
$$\|\vec{x}+\vec{y}\|^2 + \|\vec{x}-\vec{y}\|^2 = 2\|\vec{x}\|^2 + \cancel{2\vec{x}\cdot\vec{y}} - \cancel{2\vec{x}\cdot\vec{y}} + 2\|\vec{y}\|^2$$
$$= 2\|\vec{x}\|^2 + 2\|\vec{y}\|^2. \quad \blacksquare$$

17. Suppose A, B, C are vertices of a triangle. Find $\overline{AB} + \overline{BC} + \overline{CA}$. Show enough work. If you include a picture, be sure to also include an explanation of what the picture means/says.



$\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$ by vector addition rules.

18. Find an equation of the plane passing through the points $P(1,2,3)$, $Q(4,0,-1)$, and $R(2,-4,-2)$.

$\vec{PQ} = \langle 3, -2, -4 \rangle$
$\vec{PR} = \langle 1, -6, -5 \rangle$

$\vec{n} = \vec{PQ} \times \vec{PR} = \langle 10-24, -4+15, -18+2 \rangle$
$= \langle -14, 11, -16 \rangle$

$\vec{r}_0 = \vec{Q} = \langle 4, 0, 1 \rangle$

$d = -\vec{n} \cdot \vec{r}_0 = -(-56+16)$
$= 40$

So,

$\boxed{\pi : -14x + 11y - 16z + 40 = 0}$

19. Find the distance between the given point and the given plane.

$$\begin{cases} P(-1,3,-2) \\ \Pi : 3x + 2y + 6z = 5 \end{cases}$$

$$\text{dist} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}} = \frac{|3(-1) + 2(3) + 6(-2) - 5|}{\sqrt{9+4+36}} = \frac{|-3+6-12-5|}{7}$$

$$= \frac{14}{7}$$

$$= 2$$

20. Find all three equations (vector, parametric, symmetric) of the line passing through $P(0, \frac{1}{2}, 1)$ and $Q(3, 1, -4)$.

$\vec{v} = \vec{PQ} = \langle 3, \frac{1}{2}, -5 \rangle$

vector: $\vec{r}(t) = \langle 3t, \frac{1}{2} + \frac{1}{2}t, 1 - 5t \rangle$

symmetric: $\frac{x}{3} = \frac{y - \frac{1}{2}}{1/2} = \frac{z-1}{-5}$

parametric: $\begin{cases} x = 3t \\ y = \frac{1}{2} + \frac{1}{2}t \\ z = 1 - 5t \end{cases}$