Name:_____

M243: Calculus II (Spring 2018)

Instructor: Justin Ryan

Unit II Exam: Chapters 6 and 7

Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]

*Neatly write **T** if the statement is always true, and **F** otherwise.*

_____**1.** The integral $\int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx$ is convergent.

_____**2.** $\lim\limits_{\theta \to \frac{\pi}{2}^-} \frac{1 - \cos\theta}{\sin\theta} = \lim\limits_{\theta \to \frac{\pi}{2}^-} \tan\theta$

_____**3.** Simpson's Rule gives the exact answer for $\int_{-1}^{3} \left(3x^3 - 4x^2 + 7x - 9\right) dx$.

_____**4.** If $f(x) \le g(x)$ for all $x > 0$ and $\int_{1}^{\infty} f(x)\,dx$ is convergent, then $\int_{1}^{\infty} g(x)\,dx$ is also convergent.

_____**5.** $\arctan\left(\tan(\theta)\right) = \theta$ for all $\theta \in \mathbb{R}$.

Part II: Conceptual Problems [10 points each]

Complete all 3 problems in the space provided. Show <u>enough</u> work, and write your work in a clear, organized fashion.

6. Derive the formula $\dfrac{d}{dx}\left[\arctan(x)\right] = \dfrac{1}{x^2 + 1}$.

7. Write the form of the partial fraction decomposition for the rational function

$$\frac{x^2 - 1}{(x-3)^2(x^2+2x+4)}.$$

Do **NOT** solve for the unknown coefficients.

8. You wish to evaluate the integral

$$\int \frac{1}{\sqrt{x^2 - 4x - 21}}\,dx$$

by using a trig substitution. Clearly indicate what substitution you make and write down the new integral. Do **NOT** evaluate the integral.

Part III: Computational Problems [15 points each]

Complete 4 of the 5 problems in the space provided. Show enough work. Clearly mark the one problem that you wish to OMIT.

9. Evaluate the integral. Show *enough* work.

$$\int \frac{x^2}{x^2 - 4x + 3}\, dx$$

10. Evaluate the integral. Show *enough* work.

$$\int e^{2x} \sin(x)\, dx$$

11. Determine whether the integral converges or diverges. If it converges, compute its exact value. Be sure to treat the improper integral properly.

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + 1}\, dx$$

12. Evaluate the integral. Show *enough* work.

$$\int_0^2 \frac{1}{\sqrt{4t^2 + 16}}\, dt$$

13. Evaluate the integral. Show *enough* work.

$$\int \sin^2 \theta \cos^2 \theta \, d\theta$$