Name:______________________________

M243: Calculus II (Spring 2018)

Instructor: Justin Ryan

Unit III Exam (Take Home): Chapters 8 and 9

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 10 points each.

1. Use Pappus's Theorem to compute the volume of a right circular cone with height h and base radius r. You must use Pappus's Theorem to receive credit.

2. *Pareto's Law of Income* states that the number of people with incomes between $x = a$ and $x = b$ is $N = \int_a^b Ax^{-k}\,dx$, where A and k are constants with $A > 0$ and $k > 1$. The average income of these people is

$$\bar{x} = \frac{1}{N}\int_a^b Ax^{1-k}\,dx.$$

Calculate $\bar{x}$.

3. The function

$$f(x) = \frac{e^{3-x}}{(1+e^{3-x})^2}$$

is an example of a *logistic distribution.*

a.) Verify that f is a probability density function.

b.) Find $P(3 \leq X \leq 4)$.

c.) Graph f (Use a computer). What do the mean and median appear to be? Use a computer to compute both.

4. Let c be a positive number. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

where k is a positive constant, is called a *doomsday equation* because the exponent in the expression ky^{1+c} is larger than the exponent 1 for natural growth.

a.) Determine the solution that satisfies the initial condition $y(0) = y_0$.

b.) Show that there is a finite time $t = T$ such that $\lim_{t \to T^-} y(t) = \infty$. The time T is called *doomsday*.

c.) An especially prolific breed of rabbits has the growth term $ky^{1.01}$. If two such rabbits breed initially and after three months there are 16 rabbits, then when is doomsday?

5. Populations of aphids (A) and ladybugs (L) are modeled with a Lotka-Volterra system,

$$\begin{aligned}\frac{dA}{dt} &= 2A(1-0.0001A) - 0.01AL,\\ \frac{dL}{dt} &= -0.5L + 0.0001AL.\end{aligned}$$

a.) In the absence of ladybugs, what does the model predict about the aphids?

b.) Find the equilibrium solutions.

c.) Find an expression for $\frac{dL}{dA}$.

d.) Use a computer to draw a slope field for the differential equation in part *c*). Sketch a few phase trajectories (integral curves). What do they have in common?