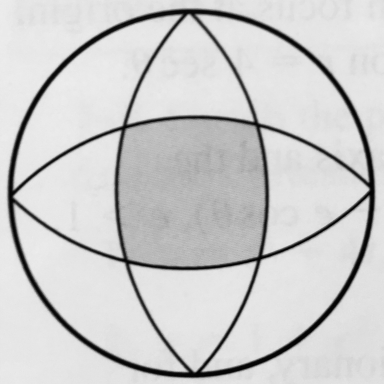
Name:_____
M243: Calculus II (Spring 2018)
Instructor: Justin Ryan
Unit IV Exam (Take Home): Chapter 10

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Consider the figure below. The outer circle has radius 1 and the centers of the interior circular arcs lie on the outer circle. Find the area of the shaded central region.



2. Show that the tangent line to the hyperbola $y = \frac{1}{x}$ touches the hyperbola halfway between the points of intersection of the tangent line and the axes.

3–5. Consider the *folium of Descartes* defined by

$$\begin{cases} x(t) = \dfrac{3t}{1+t^3}, \\ y(t) = \dfrac{3t^2}{1+t^3}. \end{cases}$$

3. *a*) Show that the curve is symmetric with respect to the line $y = x$; that is, if the point (a, b) lies on the curve, then so does (b, a). At what point does the curve intersect itself?

b) Use a computer to plot the graph of the curve. Verify that your answer to part *a*) appears to be correct.

c) Find the points on the folium of Descartes where the tangent lines are horizontal or vertical.

d) Show that the line $y = -x - 1$ is a slant asymptote of the graph.

4. *a*) Show that the Cartesian equation of the folium of Descartes is $x^3 + y^3 = 3xy$.

b) Show that the polar equation of the folium of Descartes can be written in the form

$$r = \frac{3\sec\theta\tan\theta}{1+\tan^3\theta}.$$

5. *a*) Find the area enclosed by the loop of the folium of Descartes.

b) Find the area of the region bounded between the folium of Decartes and its slant asymptote.

c) Compare your answers to parts *a* and *b*.