Name:_____
M243: Calculus II (Spring 2018)
Instructor: Justin Ryan
Unit V Exam (Take Home): Chapter 11

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{(n+3)!}$.

Hint: First write the Taylor series for e^{x+2}, then compare.

2. If $f(x) = \sin(x^3)$, find $f^{(15)}(0)$.

3. Find the sum of the series $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$.

4. Suppose $a_0 + a_1 + a_2 + \cdots + a_k = 0$. Prove that

$$\lim_{n\to\infty} \left(a_0\sqrt{n} + a_1\sqrt{n+1} + a_2\sqrt{n+2} + \cdots + a_k\sqrt{n+k} \right) = 0.$$

Hint: Try the cases $k = 1$ and $k = 2$ first, then try to deduce a general method.

5. Use the fact that $\tan(\frac{1}{2}x) = \cot(\frac{1}{2}x) - 2\cot x$ to calculate the sum of the series.

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \tan\left(\frac{x}{2^n}\right)$$