Name:_____

M243: Calculus II (Spring 2018)
Instructor: Justin Ryan
Unit V Exam: Chapter 11 (Extra Credit)

WICHITA STATE UNIVERSITY

Complete the following problems on your own paper, showing enough work. This assignment will be counted as extra credit toward your final grade. (Probably about 2–4%.) This assignment is due at the beginning of the final exam on Monday, 7 May 2018.

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1. Show that the Maclaurin series of the function

$$f(x) = \frac{x}{1-x-x^2} \quad \text{is} \quad f(x) = \sum_{n=1}^{\infty} f_n x^n$$

where f_n is the n^{th} Fibonacci number: $1, 1, 2, 3, 5, \ldots$.

Hint: Write

$$\frac{x}{1-x-x^2} = c_0 + c_1 x + c_2 x^2 + \ldots,$$

mulitply both sides of the equation by $(1-x-x^2)$, then compare powers of x.

2. Use the partial fraction decomposition of

$$f(x) = \frac{x}{1-x-x^2}$$

to find a different form of the Maclaurin series for f. Compare the coefficients to the ones that you found in problem **1** to find a non-recursive formula for the n^{th} Fibonacci number.

3. Find the Taylor series for $f(x) = \ln x$ centered at $x_0 = 1$. Plot the first 5 Taylor polynomials $(T_0, \ldots, T_5)$ on the same set of axes. [Print this and include it with your submission.] Based on the graphs, what does the radius of convergence appear to be? Compute the radius of convergence explicitly and compare with your guess.

4. The resistivity ρ of a conducting wire is the reciprocal of the conductivity and is measured in units of ohm-meters (Ωm). The resistivity of a given metal depends on the temperature according to the equation

$$\rho(t) = \rho_{20}\, e^{\alpha(t-20)} \tag{1}$$

where t is the temperature in $^\circ C$. α and ρ_{20} are constants that depend on the type of metal of the wire. In practice, when the temperature of the wire is close to $20^\circ C$, equation (1) is usually replaced by a linear or quadratic function. Find these linear and quadratic approximations. What is the maximum error of each in approximating $\rho(15)$?