Name:________________________________

M344: Calculus III (Fall 2018)

Instructor: Justin Ryan

Final Exam

Read and follow all instructions. You may not use any electronic devices. You may use a single two-sided 8.5 by 11 inch page of your own hand-written notes. Each problem is worth 22 points.

1. *a*.) Define a transformation that carries the ellipse $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ to the unit circle $u^2 + v^2 = 1$.

b.) Compute the Jacobian, $\left|\frac{\partial(x, y)}{\partial(u, v)}\right|$, of the transformation you found in part *a*.

c.) Evaluate the integral $\iint_R \sqrt{4x^2 + 9y^2}\, dA$, where R is the ellipse $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

2. *a.*) Give a parametrization of the curve C of intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z - 2x - 3y = 0$ in $\mathbb{R}^3$. Clearly state the parameter domain.

b.) Give a parametrization of the surface S given by the portion of the plane $z - 2x - 3y = 0$ inside of the cylinder $x^2 + y^2 = 9$. Clearly state the parameter domain.

c.) Let $\mathbf{F}(x, y, z) = \langle z, y, x \rangle$. Use your favorite method to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve in part *a*.

3. Consider the vector field $\mathbf{F}(x,y,z) = \langle e^y - yze^{xy}, -xze^{xy} + xe^y + e^z, ye^z - e^{xy} \rangle$.

a.) Show that $\mathbf{F}$ is conservative.

b.) Find a potential function for $\mathbf{F}$.

c.) Find the work done by the vector field $\mathbf{F}$ on a partical that moves from the point $P(1,0,0)$ to the point $Q(0,0,1)$.

4. Let f and g be functions in $\mathbb{R}^2$, both of whose second partial derivatives are continuous.

a.) Show that $\Delta(fg) = f\Delta(g) + g\Delta(f) + 2(\nabla f)\cdot(\nabla g)$.

b.) Recall that $d\mathbf{n} = \langle -dy, dx\rangle$. Show that if $\Delta f = 0$ on a simple closed region D, then

$$\int_{\partial D} (\nabla f)\cdot d\mathbf{n} = 0,$$

where ∂D denotes the positively-oriented boundary curve of the region D.

5. Consider the vector field $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$.

Use your favorite method to compute the flux, $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the surface of the solid bounded by the upper half sphere $z = 3 + \sqrt{1 - x^2 - y^2}$, the cylinder $x^2 + y^2 = 1$, and the disk $x^2 + y^2 \leq 1, z = 0$.

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