**Name**:______________________________

**M344: Calculus III** (Fall 2018)

Midterm Exam

*Read and follow all instructions. You may not use any electronic devices, but you may use one 3×5 in*$^2$ *index card of your own hand-written notes.*

**Instructions**

*Complete all problems, showing enough work. Partial credit will be given when deserved. Clearly mark your final answers, when appropriate.*

**1.** Find an equation of the plane through the points $P(1, 2, 3)$, $Q(-1, 0, 1)$, and $R(1, 1, -1)$.

**2.** Find a parametrization of the curve of intersection of the cylinder $x^2 + y^2 = 9$ and the plane $x - z = 1$.

**3.** Show that the curvature of a circle of radius $a > 0$ is $\kappa = \frac{1}{a}$.

**4.** Compute the curvature of the twisted cubic $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $P(-1, 1, -1)$.

**5.** Let $\mathbf{r}$ be a smooth space curve such that $\ddot{\mathbf{r}}(t) \neq 0$ for all $t$ in the domain of $\mathbf{r}$. Prove that $\dot{\mathbf{T}}(t) \perp \mathbf{T}(t)$ for all $t$ in the domain of $\mathbf{r}$.

**6.** Compute the second directional derivative $D_{\mathbf{v}}^2 f(x, y) = D_{\mathbf{v}}[D_{\mathbf{v}} f(x, y)]$ for $f(x, y) = x^3 + 5x^2y + y^3$ in the direction of the vector $\mathbf{v} = \langle 3, 4 \rangle$ and evaluate it at the point $(3, 2)$.

**7.** Prove the theorem: Suppose $f$ is a differentiable function of at least 2 variables. The maximum value of the directional derivative $D_{\mathbf{u}}f(\mathbf{x})$ is $\|\nabla f(\mathbf{x})\|$ and it occurs when $\mathbf{u}$ has the same direction as the gradient vector $\nabla f(\mathbf{x})$.

**8.** Let $f = f(x, y)$ be a smooth function and suppose $x = r\cos\theta$, $y = r\sin\theta$. Use the chain rule to write an expression for $\dfrac{\partial^2 f}{\partial r \partial \theta}$ in terms of the $x$- and $y$- partial derivatives of $f$.

**9.** Consider the helix $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, -t\rangle$. Find the unit tangent, normal, and binormal vectors at $t = \pi$.

**10.** Find the absolute maximum and absolute minimum values of the function $f(x, y) = 9 - x^2 - 2x - y^2 + 4y$ on the domain $x^2 + y^2 \leq 25$. You may leave your answers as reduced fractions.