Name:		
M344:	Calculus III (Fall	2018)



Midterm Exam

Read and follow all instructions. You may not use any electronic devices, but you may use one 3×5 in 2 index card of your own hand-written notes.

Instructions

Complete all problems, showing enough work. Partial credit will be given when deserved. Clearly mark your final answers, when appropriate.

1. Find an equation of the plane through the points P(1,2,3), Q(-1,0,1), and R(1,1,-1).

2. Find a parametrization of the curve of intersection of the cylinder $x^2 + y^2 = 9$ and the plane x - z = 1.

3. Show that the curvature of a circle of radius a > 0 is $\kappa = \frac{1}{a}$.

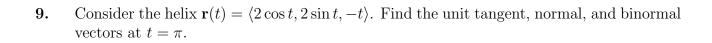
4. Compute the curvature of the twisted cubic $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point P(-1, 1, -1).

5. Let **r** be a smooth space curve such that $\ddot{\mathbf{r}}(t) \neq 0$ for all t in the domain of **r**. Prove that $\dot{\mathbf{T}}(t) \perp \mathbf{T}(t)$ for all t in the domain of **r**.

6. Compute the second directional derivative $D^2_{\mathbf{v}}f(x,y) = D_{\mathbf{v}}[D_{\mathbf{v}}f(x,y)]$ for $f(x,y) = x^3 + 5x^2y + y^3$ in the direction of the vector $\mathbf{v} = \langle 3, 4 \rangle$ and evaluate it at the point (3,2).

7. Prove the theorem: Suppose f is a differentiable function of at least 2 variables. The maximum value of the directional derivative $D_{\mathbf{u}}f(\mathbf{x})$ is $\|\nabla f(\mathbf{x})\|$ and it occurs when \mathbf{u} has the same direction as the gradient vector $\nabla f(\mathbf{x})$.

8. Let f = f(x, y) be a smooth function and suppose $x = r \cos \theta$, $y = r \sin \theta$. Use the chain rule to write an expression for $\frac{\partial^2 f}{\partial r \partial \theta}$ in terms of the x- and y- partial derivatives of f.



10. Find the absolute maximum and absolute minimum values of the function $f(x,y) = 9 - x^2 - 2x - y^2 + 4y$ on the domain $x^2 + y^2 \le 25$. You may leave your answers as reduced fractions.