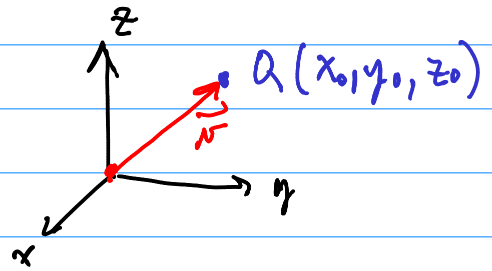
Math 344 – Calc III

8/21

Ch 12 – Review §12.1–12.4

A **vector** is a quantity that has magnitude and direction. Think "arrows"!

Let $\vec{v}$ be a vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ is said to be in **standard position** if its initial point is the origin.



We write the component form of $\vec{v}$ as

$$\vec{v} = \langle x_0, y_0, z_0 \rangle$$

Scalar Mult.: $\alpha \in \mathbb{R}$, $\quad \alpha\vec{v} = \alpha\langle x_0, y_0, z_0\rangle$

$$= \langle \alpha x_0, \alpha y_0, \alpha z_0 \rangle \qquad \text{Component-wise}$$

Vector Addition: $\vec{v} = \langle x_0, y_0, z_0 \rangle \quad \vec{w} = \langle x_1, y_1, z_1 \rangle$

$$\vec{v} + \vec{w} = \langle x_0 + x_1, y_0 + y_1, z_0 + z_1 \rangle$$

Dot Product: $\vec{v} = \langle x_0, y_0, z_0 \rangle \quad \vec{w} = \langle x_1, y_1, z_1 \rangle$

$$\vec{v} \cdot \vec{w} = x_0 \cdot x_1 + y_0 \cdot y_1 + z_0 \cdot z_1 \qquad \text{a real \#!}$$

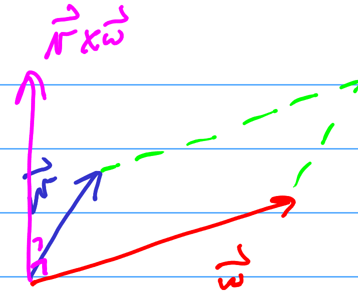
The **Length** of a vector $\vec{v}$ is $\quad \|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

The **distance** between two vectors $\vec{v}, \vec{w}$, $\quad d(\vec{v}, \vec{w}) = \|\vec{v} - \vec{w}\|$

The **angle** between $\vec{v}$ and $\vec{w}$ is $\quad \theta = \cos^{-1}\left(\dfrac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}\right)$

$$0 \leq \theta \leq \pi$$

<u>Cross Product</u> only in $\mathbb{R}^3$!



$\vec{v} = \langle x_0, y_0, z_0\rangle \qquad \vec{w} = \langle x_1, y_1, z_1\rangle$

$$\vec{v}\times\vec{w} = \begin{array}{c} \begin{matrix} + & \quad - & \quad + \end{matrix} \\ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_0 & y_0 & z_0 \\ x_1 & y_1 & z_1 \end{vmatrix} \end{array} = \vec{i}\,\underline{(y_0 z_1 - y_1 z_0)} - \vec{j}\,(x_0 z_1 - z_0 x_1) + \vec{k}\,(x_0 y_1 - y_0 x_1)$$

$$\vec{v}\times\vec{w} = \langle y_0 z_1 - y_1 z_0,\ z_0 x_1 - x_0 z_1,\ x_0 y_1 - y_0 x_1\rangle$$

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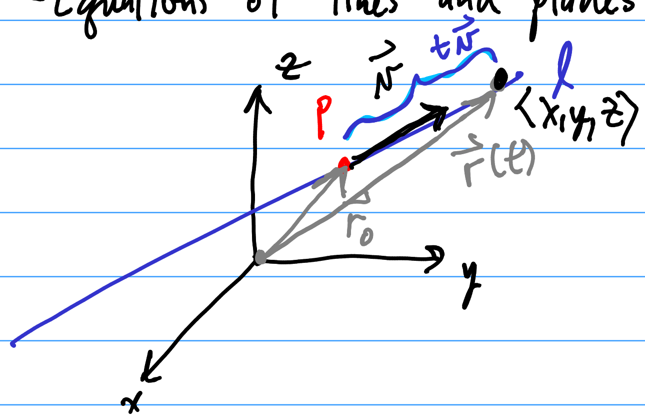
§12.5 - Equations of lines and planes in space



$\vec{r}_0$ is the pos. vector for P.

if $P(x_0, y_0, z_0)$

$\vec{r}_0 = \langle x_0, y_0, z_0\rangle$

$\vec{v} = \langle a, b, c\rangle$

Let $t \in \mathbb{R}$ be a parameter. Then the <u>vector equation</u> of ℓ is

$$(*) \quad \boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}}$$

Plug in comp. forms of $\vec{r}_0$ and $\vec{v}$

$$\vec{r}(t) = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$$
$$= \langle x_0, y_0, z_0\rangle + \langle ta, tb, tc\rangle$$

$$\boxed{\vec{r}(t) = \langle x_0 + at,\ y_0 + bt,\ z_0 + ct\rangle}$$

$\langle x,y,z\rangle = \langle x_0+at,\ y_0+bt,\ z_0+ct\rangle$

Parametric Eq'ns of ℓ:

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \quad \leftarrow x = x(t)$$

solve each eq'n for t: $\quad t = \boxed{\dfrac{x-x_0}{a} = \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c}}$

Symmetric Eq'ns of ℓ

Ex. Find all 3 eq'ns of the line thru $P(5,1,3)$ in the direction $\vec{v} = \vec{i} + 4\vec{j} - 2\vec{k}$.

$\vec{r}_0 = \langle 5,1,3\rangle$ $\qquad \vec{v} = \langle 1,4,-2\rangle$

$x_0, y_0, z_0 \qquad\qquad a, b, c$

vector: $\vec{r}(t) = \langle 5+t,\ 1+4t,\ 3-2t\rangle$

Parametric: $\begin{cases} x = 5+t \\ y = 1+4t \\ z = 3-2t \end{cases}$ $\qquad$ Symm: $\boxed{\dfrac{x-5}{1} = \dfrac{y-1}{4} = \dfrac{z-3}{-2}}$

Ex. At what point does the line through $A(2,4,-3)$ and $B(3,-1,1)$ intersect the xy-plane?

Step 1. Find an eq'n of ℓ. $\quad \vec{v} = \overrightarrow{AB} = \langle 1,-5,4\rangle$

$\vec{r}_0 = \langle 2,4,-3\rangle$

Parametric:
$$\begin{cases} x = 2 + t \\ y = 4 - 5t \\ z = -3 + 4t \end{cases}$$

Step 2. Solve $0 = -3 + 4t$ for t.

$$t = 3/4$$

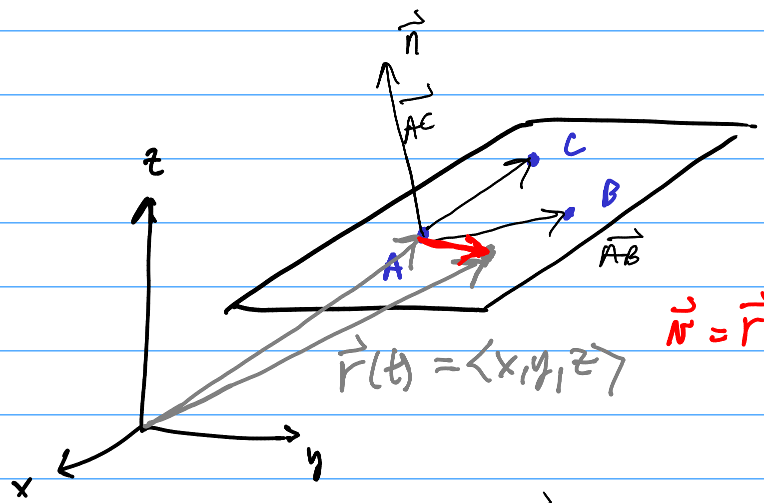
Step 3. $x(3/4) = 2 + 3/4 = 11/4$

$y(3/4) = 4 - 5(3/4) = 1/4$

The intersection pt is $(11/4, 1/4, 0)$

Planes.

Need 3 non-collinear pts in the plane.



Π = the plane

$\vec{n} = \vec{r} - \vec{r}_0$ where $\vec{r}_0$ = pos. of A

$\vec{n}$ is the normal vector to Π

$\vec{n} = \vec{AB} \times \vec{AC}$

Suppose $\vec{n} = \langle a, b, c \rangle$ $\quad A(x_0, y_0, z_0)$

RE. Finish it!