M344 9/11/18

Ch13 Exam opens today @ 12:15 pm

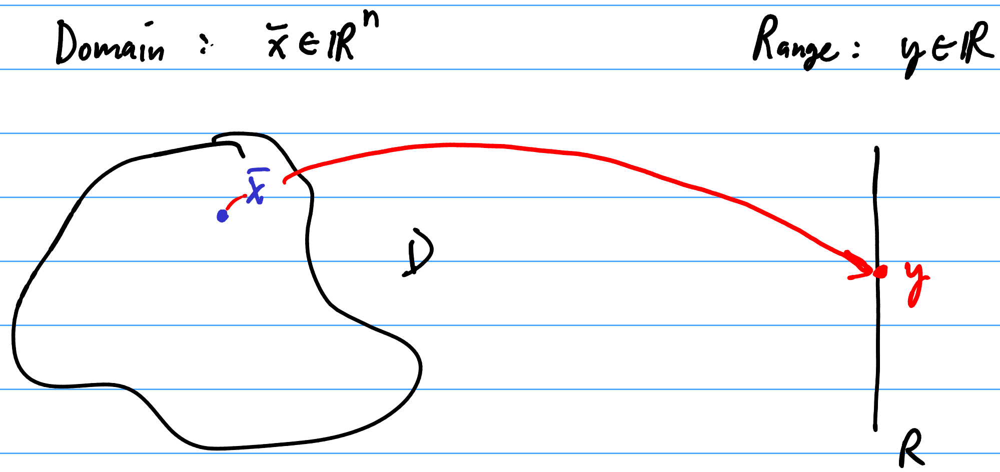
closes Friday @ 11:59 pm

Ch 14 – Partial Derivatives

§14.1 – Functions of multiple variable

A function is a relation between two sets (domain, D, and range, R) such that for each $x \in D$ there corresponds exactly one $y = f(x) \in R$.

Domain : $\bar{x} \in \mathbb{R}^n$ Range : $y \in \mathbb{R}$



$f(\bar{x}) = z$ or $f(x,y) = z$

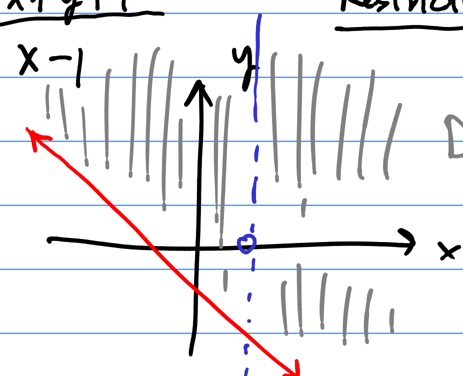
Ex. $f(x,y) = \sqrt{x+y+1}$ Restriction : $x+y+1 \geq 0$

$\boxed{y \geq -x-1}$

or $\boxed{-y \leq x+1}$

$\boxed{x \neq 1}$

Domain :

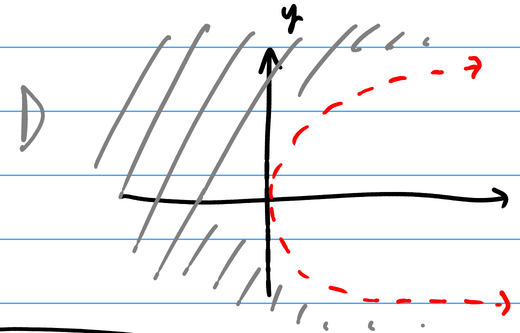


Ex. $f(x,y) = x\ln(y^2 - x)$

$y^2 - x > 0$

$y^2 > x$ Restriction

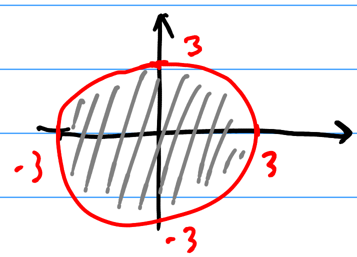
D



Ex. $g(x,y) = \sqrt{9 - x^2 - y^2}$

Domain: $9 - x^2 - y^2 \geq 0$

$x^2 + y^2 \leq 9$



Range: $z = \sqrt{9 - x^2 - y^2}$

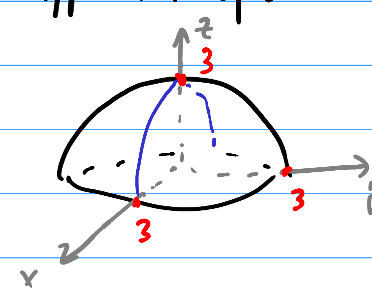
$0 \leq z \leq 3$

Graph?

$z^2 = 9 - x^2 - y^2$

$x^2 + y^2 + z^2 = 9$

upper-half-sphere

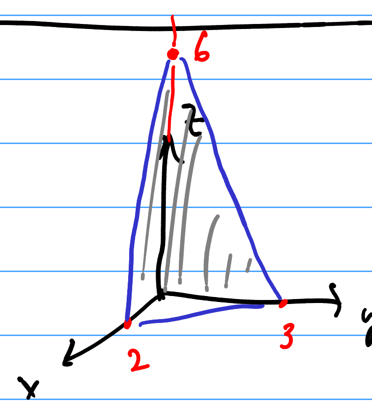


Ex. $F(x,y) = 6 - 3x - 2y$

Domain? All of $\mathbb{R}^2$.

Graph? $z = 6 - 3x - 2y$

$3x + 2y + z - 6 = 0$ Plane!



Ex. $h(x,y) = 4x^2 + y^2$

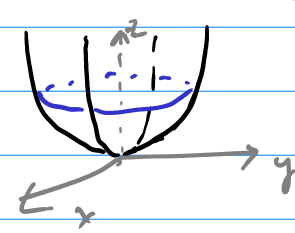
Domain? $\mathbb{R}^2$

Range: $[0, \infty)$

Graph:

$z = 4x^2 + y^2$

Elliptic Paraboloid:

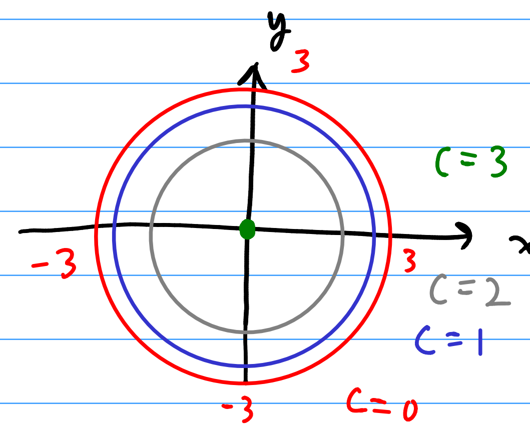


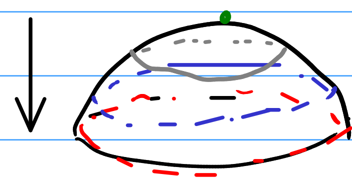
Def'n. Given a function $z=f(x,y)$. The <u>level curves</u> of f are the 2D curves in the xy-plane whose (implicit) equations are $f(x,y)=C$.

"Contour Maps"

Ex. $g(x,y)=\sqrt{9-x^2-y^2}$ semi-sphere.

C	$g(x,y)=C$		
0	$\sqrt{9-x^2-y^2}^{\,2}=0^2$	$\longrightarrow$	$x^2+y^2=9$
1	$9-x^2-y^2=1^2$	$\longrightarrow$	$x^2+y^2=8$
2	$9-x^2-y^2=4$	$\longrightarrow$	$x^2+y^2=5$
3	$9-x^2-y^2=9$	$\longrightarrow$	$x^2+y^2=0$



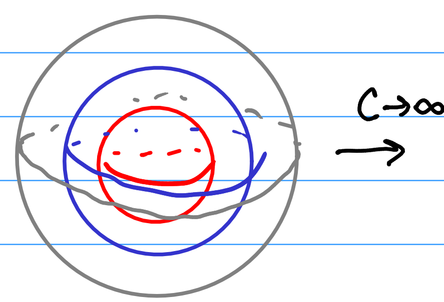


Ex. $F(x,y,z)=w$ level <u>surfaces</u>.

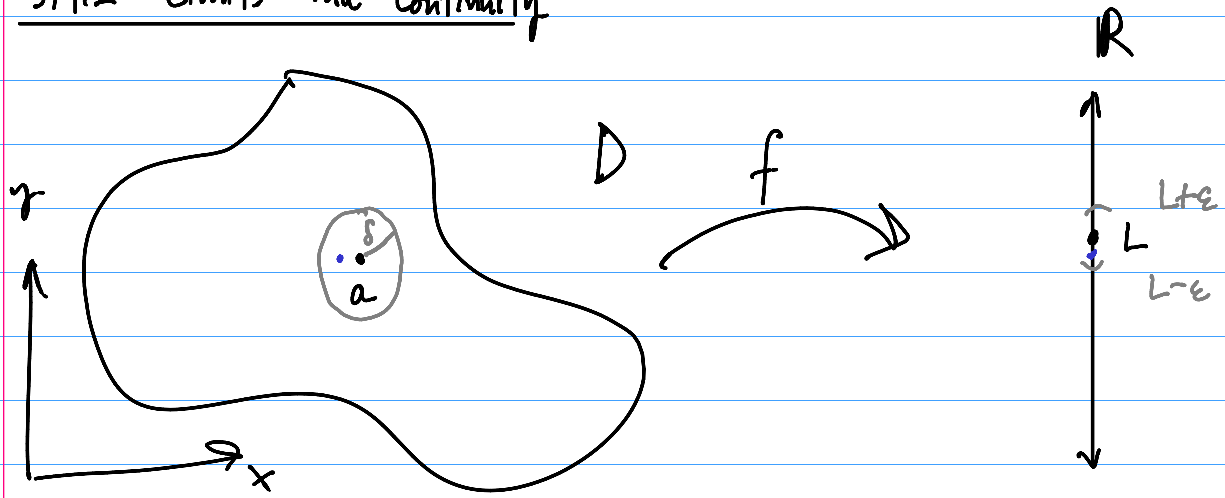
$F(x,y,z)=x^2+y^2+z^2$ Range? $[0,\infty)$

$F(x,y,z)=C^2$

Level surfaces: $x^2+y^2+z^2=C^2$



§14.2 – Limits and Continuity



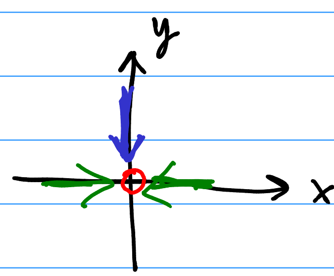
Defn. Let f be a function of two variables, $\vec{x} = (x,y)$, whose domain contains arbitrarily close to $\vec{a} = (a,b)$ in all directions. Then we say the limit of f as $\vec{x}$ approaches $\vec{a}$ is L if and only if

for each $\varepsilon > 0$, there exists a corresponding $\delta > 0$ such that

if $0 < \|\vec{x} - \vec{a}\| < \delta$, then $|f(\vec{x}) - L| < \varepsilon$.

We write $\displaystyle\lim_{\vec{x} \to \vec{a}} f(\vec{x}) = L$ or $\displaystyle\lim_{(x,y) \to (a,b)} f(x,y) = L$.

Ex. $\displaystyle\lim_{\vec{x} \to \vec{0}} \frac{x^2 - y^2}{x^2 + y^2} = \text{DNE}$



$$\Downarrow \quad \lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

1. fix $x = 0$: $\displaystyle\lim_{y \to 0} \frac{0^2 - y^2}{0^2 + y^2} = \lim_{y \to 0} \frac{-\cancel{y^2}}{\cancel{y^2}} = \lim_{y \to 0} -1 = -1$

2. fix $y = 0$: $\displaystyle\lim_{x \to 0} \frac{x^2 - 0^2}{x^2 + 0^2} = \lim_{x \to 0} \frac{x^2}{x^2} = 1$

$\Big\} \neq$

Ex. $f(x,y) = \dfrac{xy^2}{x^2+y^4}$ $\qquad \lim_{\vec{x}\to\vec{0}} f(\vec{x}) = \text{DNE}$

1. $x=0$: $\lim_{y\to 0} \dfrac{0y^2}{0^2+y^4} = \lim_{y\to 0} \dfrac{0}{y^4} = 0$

2. $y=0$: $\lim_{x\to 0} \dfrac{x\cdot 0^2}{x^2+0^4} = \lim_{x\to 0} \dfrac{0}{x^2} = 0$

3. $\underline{y=x}$: $\boxed{\lim_{x\to 0} \dfrac{x^3}{x^2+x^4} = \dfrac{0}{0} \overset{(L'H)^3}{=} \lim_{x\to 0} \dfrac{6}{24x} = \pm\infty}$ ← WRONG !!

4. $x=y^2$ $\lim_{y\to 0} \dfrac{y^2y^2}{(y^2)^2+y^4} = \lim_{y\to 0} \dfrac{y^4}{2y^4} = \dfrac{1}{2}$

$$\lim_{x\to 0} \frac{x^3}{x^2+x^4} \overset{(L'H)^2}{=} \lim_{x\to 0} \frac{6x}{2+12x^2} = \frac{0}{2+0} = 0$$

OR

$$\lim_{x\to 0} \frac{x^3}{(x^2+x^4)} \frac{1/x^2}{1/x^2} = \lim_{x\to 0} \frac{x}{1+x^2} = \frac{0}{1+0^2} = 0$$

So we must use the path $x=y^2$ to show the limit is DNE.