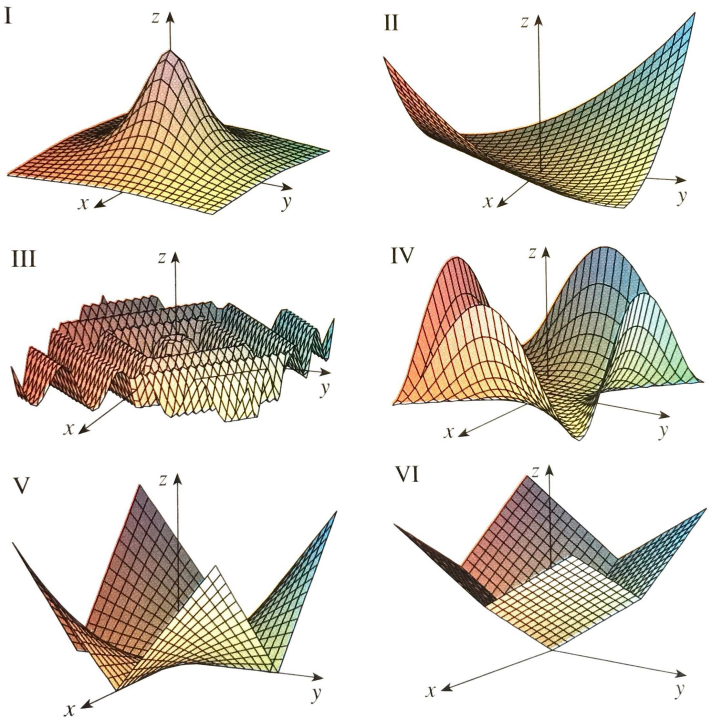
3. Functions of Several Variables

These Good Problems cover material from sections 13.1 and 13.2 of our book. Topics include functions of two variables, their contour maps, and limits.

1. Match the function with its graph.



_____ *a.)* $f(x,y) = |x| + |y|$ _____ *b.)* $f(x,y) = |xy|$

_____ *c.)* $f(x,y) = \dfrac{1}{1+x^2+y^2}$ _____ *d.)* $f(x,y) = (x^2-y^2)^2$

_____ *e.)* $f(x,y) = (x-y)^2$ _____ *f.)* $f(x,y) = \sin(|x| + |y|)$

2. Find and sketch the domain of the function

$$F(x, y) = \arcsin(x^2 + y^2 - 2).$$

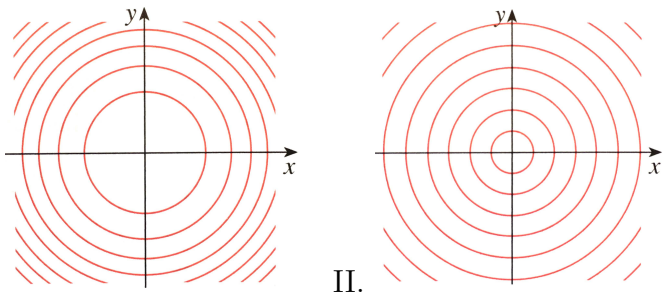
3. Find and sketch the domain of the function

$$G(x, y) = \ln(9 - x^2 - 9y^2).$$

4. Find and sketch the domain of the function

$$f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}.$$

5. Two contour maps are given. One is for a function f whose graph is a cone. The other is for a function g whose graph is a paraboloid. Which is which, and why?



I. II.

6. A thin metal plate, located in the xy-plane, has temperature $T(x, y)$ at the point (x, y). The level curves of T are called *isothermals* because at all points on such a curve the temperature is the same. Sketch some isothermals if the temperature function is given by

$$T(x, y) = \frac{100}{1 + x^2 + 2y^2}.$$

7. Use a computer to investigate the family of functions

$$f(x, y) = e^{cx^2+y^2}.$$

How does the shape of the graph depend on c?

8. Show that the limit does not exist, $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$.

9. Use polar coordinates to find the limit. (You may assume the limit exists.)

$$\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln(x^2 + y^2)$$

10. Use a computer graph of the function to explain why the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$$

11. Use the ε-δ definition of limit to prove that the limit exists.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{2x^2 + 2y^2} = 0$$