7. Double Integrals

This week's Good Problems cover material from sections 14.1 and 14.2 of our book. Topics include the integration of functions of two variables over regions in the xy-plane.

1. Evaluate the double integrals by first identifying them as the volume of solids.

a.) $\displaystyle\iint_R 3\,dA, \quad R = \{(x,y) \mid -2 \le x \le 2,\ 1 \le y \le 6\}$

b.) $\displaystyle\iint_R (5-x)\,dA, \quad R = \{(x,y) \mid 0 \le x \le 5,\ 0 \le y \le 3\}$

2. The integral $\iint_R \sqrt{9-y^2}\,dA$, where $R=[0,4]\times[0,2]$ represents the volume of a solid. Sketch the solid.

3. Find the volume of the solid that lies under the hyperbolic paraboloid $z=4+x^2-y^2$ and above the square $R=[-1,1]\times[0,2]$.

4. Calculate the iterated integrals.

a.) $\displaystyle\int_1^3\int_0^1 (1+4xy)\,dx\,dy$

b.) $\displaystyle\int_0^2\int_0^{\frac{\pi}{2}} x\,\sin y\,dy\,dx$

c.) $\displaystyle\int_0^1\int_1^2 \frac{xe^x}{y}\,dy\,dx$

d.) $\displaystyle\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$

5. Calculate the double integrals.

a.) $\displaystyle\iint_R \frac{xy^2}{x^2+1}\,dA, \quad R = \{(x,y) \mid 0 \le x \le 1,\ -3 \le y \le 3\}$

b.) $\displaystyle\iint_R x\sin(x+y)\,dA, \quad R = \left[0, \tfrac{\pi}{6}\right] \times \left[0, \tfrac{\pi}{3}\right]$

c.) $\displaystyle\iint_R \frac{x}{1+xy}\,dA, \quad R = [0,1] \times [0,1]$

6. The *average value* of a function f over a rectangle R is defined to be

$$f_{avg} = \frac{1}{\text{Area}(R)} \iint_R f(x, y)\, dA.$$

Find the average value of $f(x, y) = x^2 y$ where R has vertices $(-1, 0)$, $(-1, 5)$, $(1, 5)$, and $(1, 0)$.

7. In what way are Fubini's Theorem and Clairaut's Theorem similar? If $f(x, y)$ is continuous on the rectangle $[a, b] \times [c, d]$ and

$$g(x, y) = \int_a^x \int_c^y f(u, v)\, du\, dv$$

for $a < x < b$ and $c < y < d$. Show that $\partial_x \partial_y g = \partial_y \partial_x g = f(x, y)$.

8. Evaluate the double integral

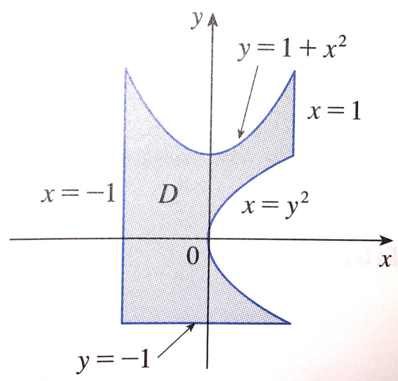
$$\iint_D xy^2 \, dA$$

where D is the region enclosed by the curves $x = 0$ and $x = \sqrt{1 - y^2}$.

9. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, and $z = 0$ in the first octant.

10. Express D as a union of a type I and type II region, then evaluate the integral

$$\iint_D xy\,dA$$



11. Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$$