Ch 14: Partial Derivatives

14.1: Functions of Several Variables.

Def'n A function f of two variables is a rule that assigns to each ordered pair of real numbers (x,y) in a set D, a unique real number denoted by $f(x,y)$.

The set D is the domain of f and its range is the set $R = \{ z \in \mathbb{R} \mid z = f(x,y) \}$.

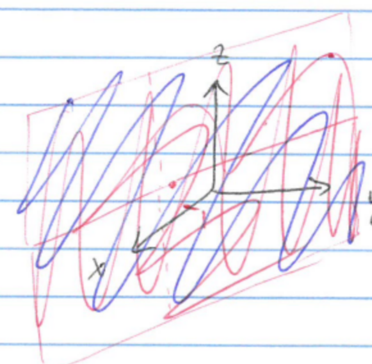
We write $z = f(x,y)$ to represent a function f

x and y are the independent variables, and z is the dependent variable.

If no domain is specified, then it is assumed to be the largest set D for which f is defined.

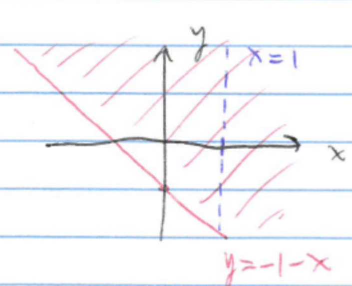
Ex. a) $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$ Evaluate $f(3,2)$ and sketch the domains

b) $f(x,y) = x \ln(y^2 - x)$

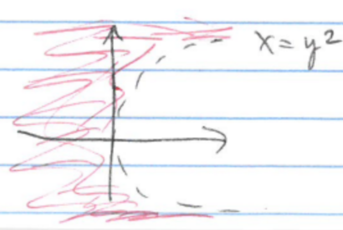


Whoops ¨

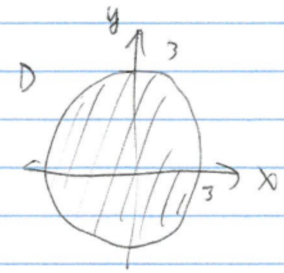
a) $x+y+1 \geq 0 \Rightarrow x+y \geq -1$
and $x \neq 1$

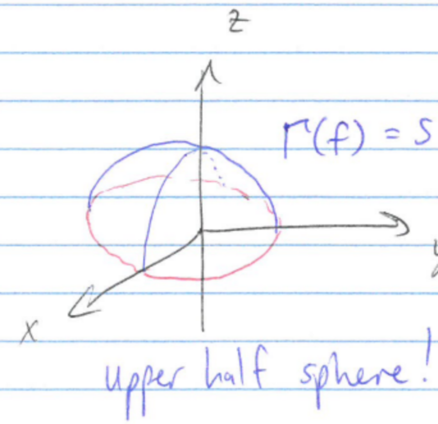


b) $y^2 - x > 0$
$y^2 > x$



Ex. Find the domain and range of $g(x,y) = \sqrt{9 - x^2 - y^2}$ and sketch the graph.

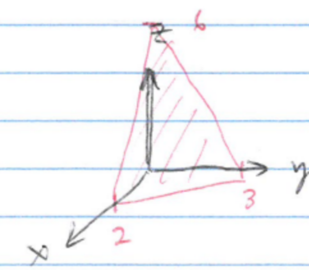




Def'n. The <u>graph</u> of a function f of two variables is the set of all points (x,y,z) in $\mathbb{R}^3$ such that $z = f(x,y)$ and $(x,y) \in D$.

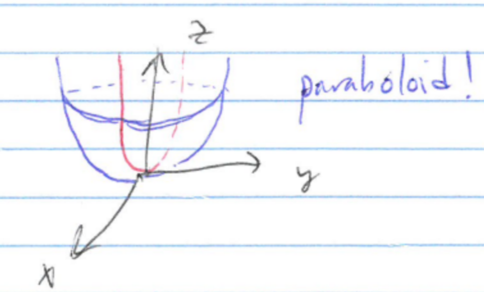
$$\Gamma(f) = \{ (x,y,z) \in \mathbb{R}^3 \mid z = f(x,y) \}.$$

Ex. Sketch the graph of $F(x,y) = 6 - 3x - 2y$.



Ex. $h(x,y) = 4x^2 + y^2$

Domain, range, graph



<u>Level curves</u>

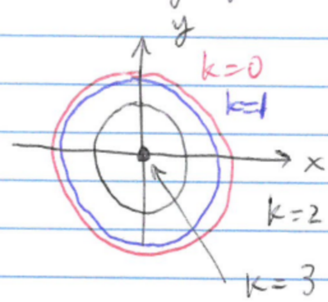
Def'n. The <u>level curves</u> of a function f of two variables are the curves w/ equation $f(x,y) = k$, where $k \in \mathrm{Range}(f)$ is a constant.

Level curves, cont'd

Def'n. The level curves of a function $f(x,y)=z$ are the curves w/ equation $f(x,y)=k$ in the $z=k$ plane. $k \in \text{Range}(f)$.

- Contour maps

Ex. Sketch the level curves of $g(x,y)=\sqrt{9-x^2-y^2}$



Ex. $h(x,y)=4x^2+y^2+1$

sketch some level curves — or at least identify the shapes.

* Notice that the level curves are just the z-traces of the surface $S=\Gamma(h)$.

Functions of Three Variables

A function of three variables is a rule assigning to each ordered triple $(x,y,z) \in D \subset \mathbb{R}^3$ a unique real number $f(x,y,z) \in \mathbb{R}$.

Ex. Find the domain of $f(x,y,z)=\ln(z-y)+xy\sin z$.

- It's very difficult to visualize the graphs of functions of 3 variables because they live in $\mathbb{R}^4$.

Here the level surfaces become extremely useful.

Ex. Find the level surfaces $f(x,y,z)=k$ (or k^2) of the function

$$f(x,y,z) = x^2+y^2+z^2$$

Extend to functions of many variables.

Three points of view:

1. $f = f(x_1, x_2, \ldots, x_n)$ is a function of n real variables.

2. $f = f((x_1, x_2, \ldots, x_n)) = f(p)$ where $p = (x_1, x_2, \ldots, x_n)$ is a function of single points in $\mathbb{R}^n$.

3. $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$, $f(\langle x_1, x_2, \ldots, x_n \rangle) = f(\vec{x})$ is a function of a single vector in $\mathbb{R}^n$.

All 3 points of view are useful, but especially the last!