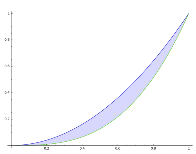
Name:______________________________
M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Unit III Exam: Chapter 15 (In class)

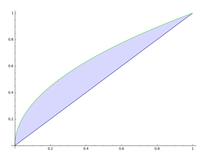
Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

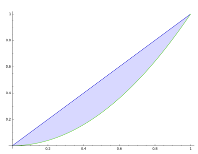
Multiple Choice [5 points each]
Select the best answer and write the corresponding letter on the line provided.

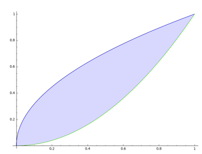
_______**1.** The double integral describes the area of which region?

$$\int_0^1 \int_{x^2}^{x} dy\, dx$$

A. 

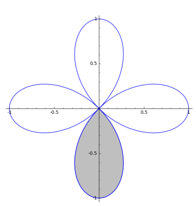
B. 

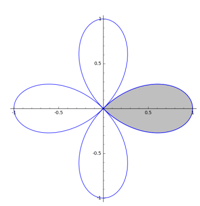
C. 

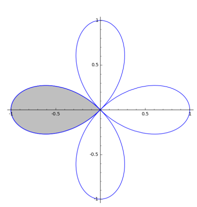
D. 

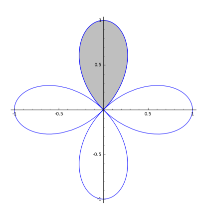
_______**2.** The double integral describes the area of which region?

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\cos(2\theta)} r\, dr\, d\theta$$

A. 

B. 

C. 

D. 

_______**3.** Compute the (determinant of the) Jacobian of the transformation

$$T : (u, v) \mapsto (\sin u, \cos v).$$

A. 1

B. $\sin u \sin v + \cos u \cos v$

C. $\sin u \sin v - \cos u \cos v$

D. $-\sin u \sin v - \cos u \cos v$

_______**4.** Convert the function $y = -2x + 1$ to a polar function, $r = f(\theta)$.

A. $r = \dfrac{1}{\sin\theta + 2\cos\theta}$

B. $r = -2\tan\theta + 1$

C. $r = \dfrac{2}{\sin\theta + \cos\theta}$

D. $r = \dfrac{1}{2\sin\theta + \cos\theta}$

_______**5.** Describe the solid bounded below the paraboloid $z = 2\sqrt{x^2 + y^2}$ and sitting above the disk $x^2 + y^2 \le 4$ in cylindrical coordinates.

A. $\begin{cases} z : 0 \to 2r \\ r : 0 \to 4 \\ \theta : 0 \to 2\pi \end{cases}$

B. $\begin{cases} z : 0 \to r^2 \\ r : 0 \to 2 \\ \theta : 0 \to 2\pi \end{cases}$

C. $\begin{cases} z : 0 \to 2r \\ r : 0 \to 2 \\ \theta : 0 \to 2\pi \end{cases}$

D. $\begin{cases} z : 0 \to 2r \\ r : 0 \to 2 \\ \theta : 0 \to \pi \end{cases}$

_______**6.** Describe the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the cone $z = \sqrt{x^2 + y^2}$ with $x \geq 0$ and $y \geq 0$ in spherical coordinates.

A. $\begin{cases} \rho : 0 \to 1 \\ \varphi : 0 \to \frac{\pi}{4} \\ \theta : 0 \to \frac{\pi}{2} \end{cases}$ **B.** $\begin{cases} \rho : 0 \to 1 \\ \varphi : 0 \to \frac{\pi}{2} \\ \theta : 0 \to \frac{\pi}{4} \end{cases}$

C. $\begin{cases} \rho : 0 \to 1 \\ \varphi : \frac{\pi}{4} \to \frac{\pi}{2} \\ \theta : 0 \to 2\pi \end{cases}$ **D.** $\begin{cases} \rho : 0 \to 1 \\ \varphi : 0 \to \frac{\pi}{4} \\ \theta : 0 \to 2\pi \end{cases}$

_______**7.** Which integral represents the volume of a sphere of radius R?

A. $\displaystyle\int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho \sin^2 \varphi \, d\rho \, d\varphi \, d\theta$ **B.** $\displaystyle\int_0^{2\pi} \int_0^{R} \int_0^{1} dz \, dr \, d\theta$

C. $\displaystyle\int_0^{2\pi} \int_0^{R} \int_0^{1} r \, dz \, dr \, d\theta$ **D.** $\displaystyle\int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$

_______**8.** Write the function $z = x + y$ as a spherical function $\rho = \rho(\varphi, \theta)$.

A. $\rho = 1$ **B.** $\rho = \cos\theta + \sin\theta$

C. $\rho = (\cos\theta + \sin\theta)\tan\varphi$ **D.** $\rho = \tan\theta + \tan\varphi$

______**9.** Consider the region bounded between the surfaces $z = x^2 + y^2$ and $z = \sqrt{x^2 + y^2}$. Write a triple integral in cylindrical coordinates that represents the volume of the region.

A. $\displaystyle\int_0^{2\pi}\int_0^1\int_r^{r^2} r\,dz\,dr\,d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^1\int_{r^2}^{r} r\,dz\,dr\,d\theta$

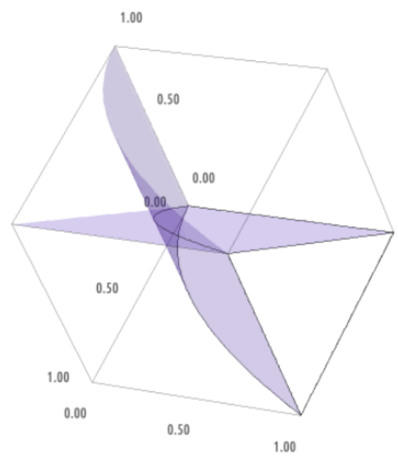
C. $\displaystyle\int_0^1\int_0^1\int_{\sqrt{x^2+y^2}}^{x^2+y^2} dz\,dy\,dx$

D. $\displaystyle\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$

______**10.** Consider the integral

$$\int_0^1\int_0^{x^2}\int_0^x f(x,y,z)\,dz\,dy\,dx$$

Change the order of integration to $dx\,dy\,dz$.



A. $\displaystyle\int_0^x\int_0^{x^2}\int_0^1 f(x,y,z)\,dx\,dy\,dz$

B. $\displaystyle\int_0^1\int_{z^2}^1\int_z^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$

C. $\displaystyle\int_0^1\int_{z^2}^1\int_{\sqrt{y}}^{z} f(x,y,z)\,dx\,dy\,dz$

D. $\displaystyle\int_0^1\int_0^z\int_0^{y^2} f(x,y,z)\,dx\,dy\,dz$

Written Problems [5 points each]

Complete all problems, showing enough work. Be sure to only do as much work as the problem asks you to do.

11. Compute the Jacobian $\left|\dfrac{\partial(x,y,z)}{\partial(u,v,w)}\right|$ of the transformation.

$$T:\begin{cases} x = u \\ y = v^2 \\ z = w^3 \end{cases}$$

12. Write a triple integral in spherical coordinates that represents the volume of the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$. Do **NOT** evaluate the integral.