Name:_____
M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Unit IV Exam: Chapter 16 (In class)

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any electronic devices. You may use a one-sided 8.5 × 11 inch page of your own hand-written notes.

Written Problems [12 points each]
Complete all 5 problems, showing enough work.

1. Evaluate the path integral

$$\int_C y^2 \, ds$$

where C is the positively oriented upper half of the unit circle: $x^2 + y^2 = 1$, $y \geq 0$.

2. Show that $\mathbf{F}$ is a conservative vector field, then find a potential function f satisfying $\nabla f = \mathbf{F}$.

$$\mathbf{F}(x, y, z) = \left\langle e^y, xe^y + e^z, ye^z \right\rangle$$

3. Evaluate the path integral using your favorite method.

$$\int_C xy^2\,dx - x^2y\,dy$$

where C is the closed curve consisting of the parabola $y = x^2$ from $(-1,1)$ to $(1,1)$, then the line segment from $(1,1)$ to $(-1,1)$.

4. Compute curl **F** and div **F** for the vector field.

$$\mathbf{F}(x, y, z) = \left\langle e^z, e^{xy} \sin z, e^{xy} \cos z \right\rangle$$

5. Define $\mathbf{n} = \langle dy, -dx \rangle$. Use Green's Theorem to evaluate the path integral

$$\int_C \mathbf{F} \cdot \mathbf{n}$$

where $\mathbf{F}(x, y) = \langle \frac{1}{2}x, \frac{1}{2}y \rangle$ and C is the unit circle.