Name:_____
M344: Calculus III (Su.19)
Midterm Exam
Chapters 13 and 14

WICHITA STATE UNIVERSITY

Instructions. Complete all problems on this paper, showing enough work. You may use one 3×5 in^2 card of your own hand-written notes. You may not use any elecronic devices.

1–5. True/False [4 points each] Write a **T** on the line if the statement is always true, and **F** otherwise. If you determine that the statement is false, you must give justification in the space provided to receive credit.

_____**1.** Suppose f is a twice continuously differentiable function. The curvature is 0 at every inflection point of the graph of $y = f(x)$.

_____**2.** Different parametrizations of the same curve result in identical tangent vectors at a given point on the curve.

_____**3.** Let f be a function of (x, y); then $\lim_{(x,y)\to(2,5)} f(x, y) = f(2,5)$.

_____**4.** Let $\mathbf{k} = \langle 0, 0, 1\rangle$, and f be a function of (x, y, z); then $D_{\mathbf{k}} f(x, y, z) = \partial_z f(x, y, z)$.

_____**5.** Let $f(x, y) = \sin x + \sin y$. Then $\left|D_{\mathbf{u}} f(x, y)\right| \leq \sqrt{2}$ for all points (x, y) and all unit vectors $\mathbf{u}$ in $\mathbb{R}^2$.

6–13. Multiple Choice [5 points each] Write the letter corresponding to the best answer on the line provided.

_____**6.** Let $f(x,y) = \ln\left(\sin^2(x) + \cos^2(y)\right)$. Compute $\dfrac{\partial f}{\partial x}$.

A. $\dfrac{2\sin x\cos x}{\sin^2 x + \cos^2 y}$ **B.** $\dfrac{1}{2\sin x\cos x}$

C. $\ln\left(2\sin x\cos x\right)$ **D.** $\dfrac{2\cos x}{\sin x + \cos^2 y}$

_____**7.** Let $g(x,y,z) = \dfrac{x^2 y}{z^3}$. Compute $\dfrac{\partial g}{\partial z}$.

A. $\dfrac{x^2 y}{3z^2}$ **B.** $\dfrac{2xy}{z^4}$

C. $\dfrac{1}{z^3}$ **D.** $\dfrac{-3x^2 y}{z^4}$

_____**8.** Let $z = x^2\sin(xy)$. Compute dz.

A. $2x\sin(y)\,dx\,dy$ **B.** $\left(2xy\sin(xy) - x^2\cos(y)\right)dx + x^2\sin(x)\,dy$

C. $\left(2x - (x+y)\cos(xy)\right)dx\,dy$ **D.** $\left(2x\sin(xy) - x^2 y\cos(xy)\right)dx - x^3\cos(xy)\,dy$

_____**9.** Let $f(x,y) = (x-1)^2 - 3(y-2)^2$. Find the linearization of f at the point $P(3,2)$.

A. $L(x,y) = 4 + 4x + y$ **B.** $L(x,y) = 4 + 4(x-3)$

C. $L(x,y) = 4(x-3)$ **D.** $L(x,y) = 4 + 2(x-1)(x-3) - 6(y-2)^2$

_____**10.** Let $f(x,y) = (x-1)^2 - 3(y-2)^2$. Find a formula for the directional derivative, $D_{\mathbf{v}}f(x,y)$, where $\mathbf{v} = \langle -12, 5\rangle$.

A. $-\dfrac{24}{13}(x-1) - \dfrac{30}{13}(y-2)$

B. $-\dfrac{12}{13}(x-1) - \dfrac{15}{13}(y-2)$

C. $-24(x-1) - 30(y-2)$

D. $-12(x-1)^2 - 15(y-2)^2$

_____**11.** Let $\mathbf{r} : \mathbb{R} \to \mathbb{R}^3$ be a smooth vector function. Which of the following formulas is <u>not</u> a valid formula for the curvature of $\mathbf{r}$?

A. $\kappa = \left\| \dfrac{d\mathbf{T}}{ds} \right\|$

B. $\kappa = \dfrac{\|\dot{\mathbf{T}}\|}{\|\dot{\mathbf{r}}\|}$

C. $\kappa = \dfrac{|\dot{\mathbf{r}} \cdot \ddot{\mathbf{r}}|}{\|\dot{\mathbf{r}}\|^3}$

D. $\kappa = \dfrac{\|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}\|}{\|\dot{\mathbf{r}}\|^3}$

12–13. Consider the function $f(x,y) = 5 + \sqrt{9 - x^2 - y^2}$.

_____**12.** What is the range of f?

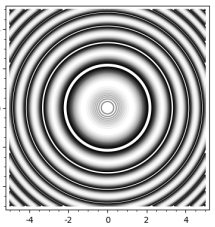
A. $[-3,3] \times [-3,3]$

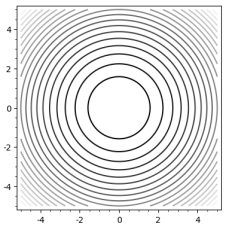
B. $[5,8]$

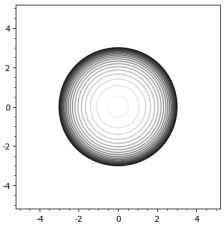
C. $[0,3]$

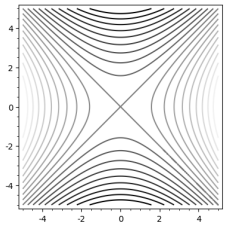
D. $\mathbb{R}$

_____**13.** Choose the graph that best represents the level curves of f.

A. 

B. 

C. 

D. 

14. [30 points] Let $\mathbf{r}$ be a smooth vector function parametrizing a space curve C. Prove that the unit tangent vector field $\mathbf{T}$ on C satisfies $\mathbf{T} \perp \dot{\mathbf{T}}$ at all points along C.

15. [30 points] Let f be a differentiable function of two variables, and let P be a point in its domain. Prove that the directional derivative $D_{\mathbf{u}} f(P)$ is maximum when $\mathbf{u}$ is in the same direction as $\nabla f(P)$, and $\max_{\|\mathbf{u}\|=\mathbf{1}} D_{\mathbf{u}} f(P) = \left\| \nabla f(P) \right\|$.

16. [40 points] Find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + 4y^2 = 16$. Leave your answers in exact form (no decimals).

17. [40 points] Find an equation of the osculating circle to the curve $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$ at $x = 2$.

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