Name:_____

M344: Calculus III (Su.19)
Good Problems 1
Sections 13.1–13.3

WICHITA STATE UNIVERSITY

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may not use any notes or electronic devices. All you need is a pencil and your brain.

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1. Consider the vector function $\mathbf{r}(t) = \left\langle \ln(2-t), \dfrac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$.

a.) What is the domain of **r**?

b.) Compute the derivative, $\dot{\mathbf{r}}(t)$.

c.) Compute the antiderivative, $\int \mathbf{r}(t)\,dt$.

2. Let $\mathbf{r}$ be a smooth vector function such that $\mathbf{r}(t) \neq \mathbf{0}$. Prove that

$$\frac{d}{dt}\Big[\|\mathbf{r}(t)\|\Big] = \frac{\mathbf{r}(t)\cdot\dot{\mathbf{r}}(t)}{\|\mathbf{r}(t)\|}.$$

Hint: $\|\mathbf{r}(t)\|^2 = \mathbf{r}(t)\cdot\mathbf{r}(t)$.

3. Find $\mathbf{r}(t)$ if $\dot{\mathbf{r}}(t) = t\,\mathbf{i} + e^t\,\mathbf{j} + te^t\,\mathbf{k}$, and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

Consider the curve C parametrized by the vector function

$$\mathbf{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle .$$

4. Find the unit tangent vector field, $\mathbf{T}(t)$, along C.

5. Find the arc length of the portion of the curve on the interval $0 \leq t \leq \frac{\pi}{4}$.

6. Reparametrize the curve

$$\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\mathbf{i} + \frac{2t}{t^2+1}\mathbf{j}$$

with respect to arc length measured from the point $(1,0)$ in the direction of increasing t. Simplify as much as possible.