

Name: _____

M344: Calculus III (Su.19)

Good Problems 2

Sections 13.3-4, 14.1



Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

1. [10 points] Show that the curvature of a circle of radius $a > 0$ is constant: $\kappa = \frac{1}{a}$.

2. [10 points] Consider a plane curve C parametrized by a vector function $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ satisfying $\dot{\mathbf{r}}(t) \neq \mathbf{0}$. Show that the curvature of C is given by

$$\kappa(t) = \frac{|\dot{x}(t)\ddot{y}(t) - \ddot{x}(t)\dot{y}(t)|}{\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}^3}.$$

Hint: Regard C as living in the xy -plane emebbed in \mathbb{R}^3 .

3. [30 points] Find an equation of the osculating circle to the curve at $x = 2$.

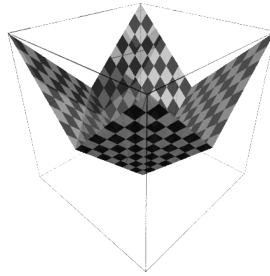
$$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$$

4. [20 points] Find the tangential and normal components of the acceleration vector. Simplify as much as possible. (You do not need to find **T** and **N**).

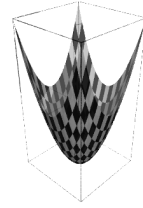
$$\mathbf{r}(t) = \langle t, 2e^t, e^{2t} \rangle$$

5. [5 points each] Match each function of 2 variables to its corresponding graph.

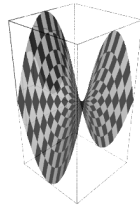
_____ *i.* $f(x, y) = \sin(x^2 + y^2)$ **A.**



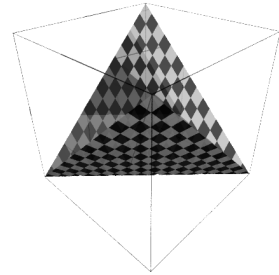
B.



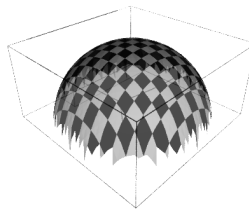
_____ *ii.* $f(x, y) = |x| + |y|$ **C.**



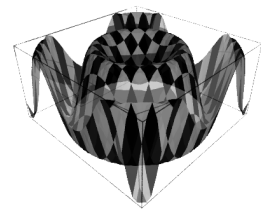
D.



_____ *iii.* $f(x, y) = x^2 + y^2$ **E.**

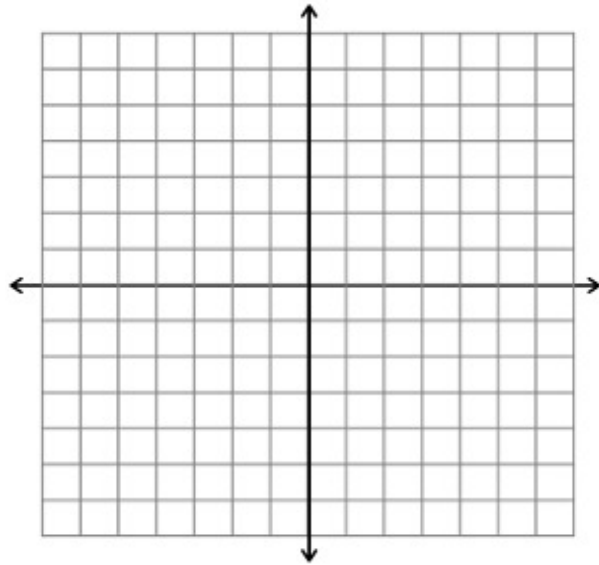


F.

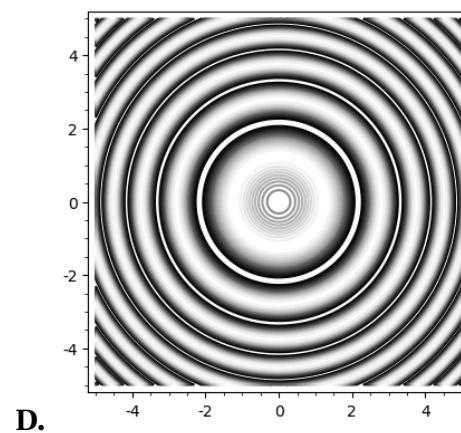
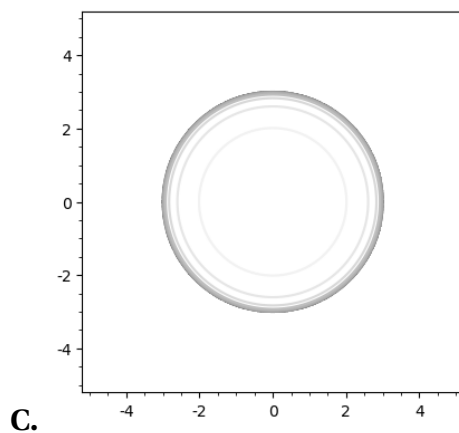
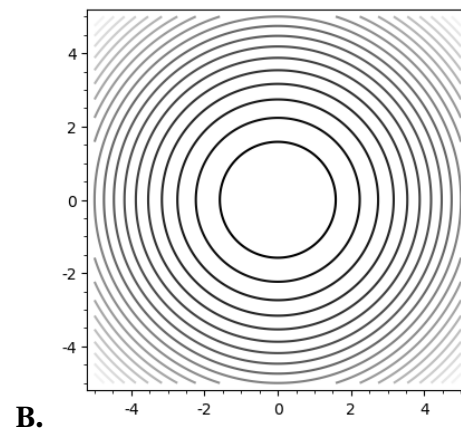
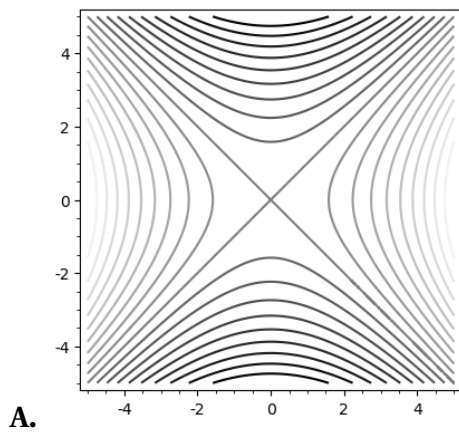


6. Consider the function $f(x, y) = \ln(9 - x^2 - y^2)$.

i. [10 points] Find and plot the domain of f . Be sure to properly label the graph.



_____ ii. [5 points] Choose the plot that best represents the level curves of f .



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