Name:________________________________

M344: Calculus III (Su.19)

Good Problems 3

Sections 14.2-14.5

WICHITA STATE UNIVERSITY

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

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1. [10 points] Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$$

2. [10 points] Use an $\varepsilon - \delta$ argument to prove that the limit does exist.

$$\lim_{(x,y)\to(0,0)} \frac{5xy^2}{x^2+y^2}$$

3. [10 points] Use polar coordinates to find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$$

Hint: In polar coordinates $r = \sqrt{x^2+y^2}$ and $r \to 0^+$ as $(x, y) \to (0,0)$.

4. [15 points] Use the limit definition of partial derivative to compute $\partial_y f$ for the function $f(x, y) = xy^2 - x^3 y$. You must use the limit definition to receive credit.

5. [15 points] Find an equation of the tangent plane to the surface $z = (x+2)^2 - 2(y-1)^2 - 5$ at the point $(2, 3, 3)$.

6. [10 points] Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

Hint: The volume of a right circular cylinder is $V(r, h) = \pi r^2 h$. Find dV.

7. [10 points] Let $F = F(x, y, z)$ be a differentiable function, and suppose $z = z(x, y)$ is an implicit function defined by the formula $F(x, y, z) = k$, where k is a constant. Show that

$$\frac{\partial z}{\partial x} = \frac{-\partial_x F}{\partial_z F}.$$

Hint: Apply the chain rule to F.

8–11. [5 points each] Compute the indicated partial derivatives of the given functions.

_______**8.** $f(x, y) = \dfrac{x}{(x+y)^2}$; Find $\partial_x f$.

A. $\dfrac{1}{(1+y)^2}$ **B.** $\dfrac{1}{2(x+y)}$

C. $\dfrac{y-x}{(x+y)^3}$ **D.** $-\dfrac{2x}{\left(x+y\right)^3}$

_______**9.** $f(x, y) = \dfrac{x}{(x+y)^2}$; Find $\partial_y f$.

A. $\dfrac{x}{2(x+y)}$ **B.** $\dfrac{x}{(x+1^2)}$

C. $\ln\left(x+y\right)$ **D.** $-\dfrac{2x}{\left(x+y\right)^3}$

_______**10.** $F(x,y) = \int_x^y \cos(e^t)\,dt$; Find $\partial_x F$.

A. $\int_1^0 \cos(e^t)\,dt$

B. $-\cos(e^x)$

C. $\int_x^y -e^t \sin(e^t)\,dt$

D. $-e^x \sin(e^x)$

_______**11.** $w = z^{y/x}$; Find $\partial_y w$.

A. $\dfrac{z^{y/x}\ln(z)}{x}$

B. $\dfrac{\ln\left(\frac{y}{x}\right)}{x}$

C. $\dfrac{y}{x} z^{y/x-1}$

D. $z^{1/x}$

12. [Bonus: 10 points] Let $a > 0$ be a constant. Show that the function

$$u(x,t) = \sin(x - at) + \ln(x + at)$$

is a solution of the wave equation,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

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