Name: M344: Calculus III (Su.19)

WICHITA STATE UNIVERSITY

Good Problems 3 Sections 14.2-14.5

**Instructions.** Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

1. [10 points] Show that the limit <u>does not</u> exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$$

2. [10 points] Use an  $\varepsilon - \delta$  argument to prove that the limit does exist.

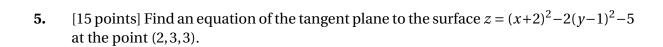
$$\lim_{(x,y)\to(0,0)} \frac{5xy^2}{x^2+y^2}$$

**3.** [10 points] Use polar coordinates to find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$$

Hint: In polar coordinates  $r = \sqrt{x^2 + y^2}$  and  $r \to 0^+$  as  $(x, y) \to (0, 0)$ .

**4.** [15 points] Use the <u>limit definition</u> of partial derivative to compute  $\partial_y f$  for the function  $f(x,y) = xy^2 - x^3y$ . You must use the limit definition to receive credit.



**6.** [10 points] Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

Hint: The volume of a right circular cylinder is  $V(r, h) = \pi r^2 h$ . Find dV.

7. [10 points] Let F = F(x, y, z) be a differentiable function, and suppose z = z(x, y) is an implicit function defined by the formula F(x, y, z) = k, where k is a constant. Show that

$$\frac{\partial z}{\partial x} = \frac{-\partial_x F}{\partial_z F}.$$

Hint: Apply the chain rule to F.

- **8–11.** [5 points each] Compute the indicated partial derivatives of the given functions.
- \_\_\_\_\_\_8.  $f(x,y) = \frac{x}{(x+y)^2}$ ; Find  $\partial_x f$ .

$$\mathbf{A.} \; \frac{1}{(1+y)^2}$$

$$\mathbf{C.} \ \frac{y-x}{(x+y)^3}$$

$$\mathbf{B.} \; \frac{1}{2(x+y)}$$

$$\mathbf{D.} - \frac{2x}{\left(x+y\right)^3}$$

**\_\_\_\_9.**  $f(x, y) = \frac{x}{(x+y)^2}$ ; Find  $\partial_y f$ .

$$\mathbf{A.} \; \frac{x}{2(x+y)}$$

**B.** 
$$\frac{x}{(x+1^2)}$$

$$\mathbf{C.} \ln (x+y)$$

$$\mathbf{D.} - \frac{2x}{\left(x+y\right)^3}$$

**\_\_\_\_\_10.**  $F(x, y) = \int_x^y \cos(e^t) dt$ ; Find  $\partial_x F$ .

$$\mathbf{A.} \int_{1}^{0} \cos(e^{t}) \, dt$$

$$\mathbf{B}_{\bullet} - \cos(e^x)$$

$$\mathbf{C.} \int_{x}^{y} -e^{t} \sin(e^{t}) dt$$

$$\mathbf{D.} - e^x \sin(e^x)$$

**\_\_\_\_11.**  $w = z^{y/x}$ ; Find  $\partial_y w$ .

$$\mathbf{A.} \, \frac{z^{y/x} \ln{(z)}}{x}$$

$$\mathbf{B.} \frac{\ln\left(\frac{y}{x}\right)}{x}$$

C. 
$$\frac{y}{x}z^{y/x-1}$$

**D.** 
$$z^{1/x}$$

**12.** [Bonus: 10 points] Let a > 0 be a constant. Show that the function

$$u(x, t) = \sin(x - at) + \ln(x + at)$$

is a solution of the wave equation,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

