Name:_____

M344: Calculus III (Su.19)
Good Problems 6
Sections 16.1–4

WICHITA STATE UNIVERSITY

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

1. [10 points] A particle moves in a velocity field $\mathbf{V}(x,y) = \langle x^2, x+y^2\rangle$. If it is at position $(2,1)$ at time $t=3$, estimate its location at time $t=3.01$.

[Hint: If $\mathbf{r}(a)$ denotes the location of the particle at time $t=a$, then $\mathbf{r}(a+dt) \approx \mathbf{r}(a)+d\mathbf{V}$, where $d\mathbf{V} = \mathbf{V}(\mathbf{r}(a))\,dt$.]

2. [15 points] Compute the path integral

$$\int_C y\,ds,$$

where C is the portion of the curve $\mathbf{r}(t) = \langle t^2, 2t\rangle$ from $(0,0)$ to $(9,6)$.

3. [25 points] Find the work done by the force field $\mathbf{F}(x, y) = \langle x, y+2 \rangle$ in moving an object along one arch of the cycloid, $\mathbf{r}(t) = \langle t - \sin t, 1 - \cos t \rangle,\ t : 0 \to 2\pi$.

4 – 6. Consider the vector field $\mathbf{F}(x,y) = \left\langle 2e^{x^2}x\sin\left(xy^2\right) + e^{x^2}y^2\cos\left(xy^2\right), 2e^{x^2}xy\cos\left(xy^2\right)\right\rangle$

4. [10 points] Show that $\mathbf{F}$ is conservative by computing and comparing $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$.

5. [5 points] Show that $f(x,y) = e^{x^2}\sin\left(xy^2\right)$ is a potential function for $\mathbf{F}$. [Hint: Compute ∇f.]

6. [10 points] Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the portion of the unit circle $x^2+y^2=1$ from $(1,0)$ to $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

7. [25 points] Use Green's Theorem to compute the area enclosed by the ellipse

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

as a path integral around the boundary. You must use Green's Theorem and a path integral to receive credit.

scratch page