| Name:   |                       |
|---------|-----------------------|
| 1/10/4/ | Colordon III (Cr. 10) |



**M344: Calculus III** (Su.19) Good Problems 6

Sections 16.1-4

**Instructions.** Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

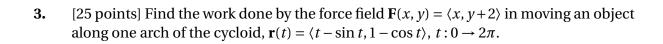
1. [10 points] A particle moves in a velocity field  $V(x, y) = \langle x^2, x + y^2 \rangle$ . If it is at position (2, 1) at time t = 3, estimate its location at time t = 3.01.

[Hint: If  $\mathbf{r}(a)$  denotes the location of the particle at time t = a, then  $\mathbf{r}(a+dt) \approx \mathbf{r}(a)+d\mathbf{V}$ , where  $d\mathbf{V} = \mathbf{V}(\mathbf{r}(a))dt$ .]

**2.** [15 points] Compute the path integral

$$\int_C y\,ds,$$

where *C* is the portion of the curve  $\mathbf{r}(t) = \langle t^2, 2t \rangle$  from (0,0) to (9,6).



- **4 6.** Consider the vector field  $\mathbf{F}(x, y) = \left\langle 2e^{x^2}x\sin\left(xy^2\right) + e^{x^2}y^2\cos\left(xy^2\right), 2e^{x^2}xy\cos\left(xy^2\right) \right\rangle$
- **4.** [10 points] Show that **F** is conservative by computing and comparing  $\frac{\partial P}{\partial y}$  and  $\frac{\partial Q}{\partial x}$ .

**5.** [5 points] Show that  $f(x, y) = e^{x^2} \sin(xy^2)$  is a potential function for **F**. [Hint: Compute  $\nabla f$ .]

**6.** [10 points] Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where C is the portion of the unit circle  $x^2 + y^2 = 1$  from (1,0) to  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ .

7. [25 points] Use Green's Theorem to compute the area enclosed by the ellipse

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

as a path integral around the boundary. You must use Green's Theorem and a path integral to receive credit.

