Name:______________________________
M344: Calculus III (Su.19)
Extra Credit: Integration
Due by: Friday, 12 July 2019 at 8:40 am

WICHITA STATE UNIVERSITY

Instructions. Complete all problems on this paper, showing enough work. You may use any resources that you'd like to complete this assignment, but be sure to write your solutions in your own words.

This assignment is optional. If submitted, it will be counted as extra credit toward your final grade.

1–4. True/False [1 point each] Write a **T** on the line if the statement is always true, and **F** otherwise. If you determine that the statement is false, you must give justification in the space provided to receive credit.

______**1.** $\displaystyle\int_0^1\int_0^x \sqrt{x+y^2}\,dy\,dx = \int_0^1\int_0^y \sqrt{x+y^2}\,dx\,dy$

______**2.** If f is continuous on $[0,1]$, then $\displaystyle\int_0^1\int_0^1 f(x)f(y)\,dy\,dx = \left[\int_0^1 f(x)\,dx\right]^2$.

______**3.** $\displaystyle\int_1^4\int_0^1 \left(x^2+\sqrt{y}\right)\sin(x^2y^2)\,dx\,dy \le 9$

______**4.** The integral $\displaystyle\int_0^{2\pi}\int_0^2\int_r^2 dz\,dr\,d\theta$ represents the volume enclosed by the cone $z = \sqrt{x^2+y^2}$ and the plane $z=2$.

5. [2 points] Compute the integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy.$

6. [2 points] Rewrite the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$ as an iterated integral in the order $dx\,dy\,dz$.

7. [2 points] Evaluate the integral $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left(x^3 + xy^2\right)\, dy\, dx$.

8. [2 points] Compute the Jacobian of the spherical coordinate transformation. Show enough work.

9. [2 points] Read about centroids and centers of mass in section 15.4 of the book.

A lamina occupies the part of the disk $x^2 + y^2 \leq a^2$, $a > 0$, that lies in the first quadrant. Find

1. the centroid of the lamina; and
2. the center of mass of the lamina if the desity function is $\rho(x, y) = xy^2$.

10. [3 points] Use a change-of-coordinates transformation to compute the integral

$$\iint_R \frac{x-y}{x+y}\, dA,$$

where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$, and $(1,3)$.

11. [3 points] Let D be a type I region in the xy-plane, and suppose f is a continuous function on D. Prove that there exists a point (x_0, y_0) in D satisfying

$$\iint_D f(x,y)\,dA = f(x_0,y_0)\cdot \text{Area}(D).$$

This is the **Mean Value Theorem** for double integrals.