Name:_____
M555: Differential Equations I (Spring 2018)
Instructor: Justin Ryan
Unit III Exam (Take Home): Chapters 5 and 6

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Use the power series method to find a fundamental set for the equation

$$y'' - 3xy' + y = 0.$$

Determine the first three terms in each of the two solutions that form the fundamental solution set.

2. Given the equation

$$(2 + x^2)y'' - xy' + 4y = 0$$

with the initial conditions $y(0) = -1$ and $y'(0) = 3$. Find the first four <u>nonzero</u> terms in the series solution, and find its radius of convergence.

3. Use the Laplace transform method to solve the initial value problem

$$\begin{cases} y'' + 2y' + 5y = 0, \\ y(0) = 2, \\ y'(0) = -1. \end{cases}$$

4. Use the Laplace transform to solve the initial value problem

$$\begin{cases} y'' + 4y = u_\pi(t), \\ y(0) = y'(0) = 0. \end{cases}$$

5. Find the solution of the initial value problem. Plot the graph of the solution and describe how the solution behaves as $x \to 0$. Include a print out of the graph with your submission.

$$\begin{cases} x^2 y'' + 3xy' + 5y = 0, \\ y(1) = 1, \\ y'(1) = -1. \end{cases}$$