Differential Equations I: Final Project

Numerical Methods for Solving First Order DE

Due: 2 May 2018

This project is worth 15% of your final grade; each part of each question is worth 10 points. Complete both problems, using Excel (or your favorite alternative) when it is appropriate. All hand-written work should be written clearly and neatly.

1. Consider the initial value problem

$$\begin{cases} y' = 2y - 3t\,; \\ y(0) = 1 \end{cases}$$

a.) Find the exact solution.

b.) Use Euler's method with $h = 0.1$ and $h = 0.05$ to find approximate values of the solution on the interval $[0, 2]$. Plot both of these approximations on the same set of axes.

c.) Use the Runge-Kutta method with the same h values to find approximate values of the solution on the interval $[0, 2]$. Plot these on the same set of axes.

d.) Calculate the error in each of the four approximations that you found by subtracting the approximate values from the values of the actual solution at each t-value. Make a dot plot of these error terms.

2. Consider the initial value problem

$$\begin{cases} y' = \dfrac{3t^2}{(3y^2 - 4)}; \\ y(0) = 0 \end{cases}$$

a.) Use the FEUT to show that a solution to this differential equation exists for this initial data.

b.) Use the Runge-Kutta method with various step sizes to estimate how far the solution can be extended to the right. Let t^* be the right endpoint of the interval of existence. What happens at t^* to prevent the solution from extending further?

c.) Solve the IVP analytically to solve for the exact value t^*. How close was your approximation? (You can use a computer to find the exact value of t^*.)

d.) The Runge-Kutta method continues to give y-values for $t > t^*$. Do these values have any significance? If so, what? If not, why?

3. Theorem. *Let $f(t,y)$ be linear in both t and y. The exact error in the k+first step of the Improved Euler Method is given by*

$$E_{k+1} = \frac{h^3 \varphi'''(t_k^*)}{6}$$

where φ is the exact solution, h is the step size, and t_k^* is some point in the interval $[t_k, t_{k+1}]$.

Consider the IVP $y' - 2y = 3t$, $y(0) = 1$. Compute the maximum error E_{10} with

a.) $h = 0.5$, and
b.) $h = 0.05$.