Name:______________________________

M511: Linear Algebra (Spring 2018)

Instructor: Justin Ryan

Unit II Exam (Take Home): Chapter 5

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Let $(V, \langle\ ,\ \rangle)$ be a finite-dimensional inner product space. Prove the Cauchy-Schwarz-Bunyachevsky Inequality:

$$|\langle \mathbf{x}, \mathbf{y} \rangle| \le \|\mathbf{x}\|\,\|\mathbf{y}\|$$

and equality holds if and only if $\mathbf{x}$ and $\mathbf{y}$ are linearly dependent.

2. Consider the vector space $\mathbb{R}^3$ with inner product defined by

$$\langle \mathbf{x}, \mathbf{y} \rangle = x_1 y_1 w_1^2 + x_2 y_2 w_2^2 + x_3 y_3 w_3^2$$

where $\mathbf{w} = (1, -2, 2)^T$. Find the matrix representation of the inner product with respect to the ordered basis

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

3. Consider the inner product space $(S, \langle\ ,\ \rangle)$ where S is the subspace of $C[0, \frac{\pi}{2}]$ spanned by the basis $\{\cos x, \cos^2 x, \cos^3 x\}$ and $\langle\ ,\ \rangle$ is the inner product

$$\langle f, g\rangle = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} f(x)g(x)\,dx.$$

Find the matrix representing $\langle\ ,\ \rangle$ with respect to the given ordered basis.

4. Consider the inner product space $(\mathbb{R}^3, \cdot)$. Use the Gram-Schmidt orthogonalization process to create an orthonormal basis from the ordered basis

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

5. Apply the Gram-Schmidt orthogonalization process to the inner product space in problem **3**.